Solución a los ejercicios de cálculo diferencial

CAPÍTULO 1

EJERCICIO 1

1. Función
2. Relación
3. Función
4. Relación
5. Función
6. Relación
7. Función
8. Relación
9. Función
10. Relación
11. Función
12. Relación
13. Función
14. Función
15. Relación

EJERCICIO 2

1. $f\left(-\frac{1}{2}\right) = -\frac{5}{2}$ $f(3) = 15, f(0) = -3$
2. $f(a) = a^2 - 5a + 6,$
   $f(a + b) = a^2 + 2ab + b^2 - 5a - 5b + 6$
   $f(x + h) = 3x^2 + 6hx + 3h^2 + 4x + 4h - 2$
3. $\frac{f(x+h)-f(x)}{h} = 6x + 3h + 4$
4. $f\left(\frac{1}{3}\right) = -\frac{1}{5}$, $f\left(-\frac{1}{2}\right)$ = No existe
   $f(x + h) - f(x) = \frac{4h}{(2x+2h+1)(2x+1)}$
5. $f(5) = 3, f(4) = 0, f(6) = \sqrt{20} = 2\sqrt{5}$,
   $f(3)$ = No está definida
6. $f(x + h) = \sqrt{x^2+2xh+h^2-3}$
   $\frac{f(x+h)-f(x)}{h} = \frac{2x+h}{\sqrt{(x+h)^2-3}+\sqrt{x^2-3}}$
7. $\frac{f(x+b)-f(x)}{b} = -\frac{1}{(x+b+1)(x+1)}$
8. $\frac{f(x+h)-f(x)}{h} = -\frac{1}{\sqrt{1-(x+h)}+\sqrt{1-x}}$
9. $f(1) = \frac{4}{3}$, $f(0) = \frac{5}{2}$, $f(x + 5) = \frac{|x|}{x+7}$
10. $f(-1) = 2, f\left(\frac{1}{x}\right) = -\frac{3}{x^2} + 2x^2 - 3x$

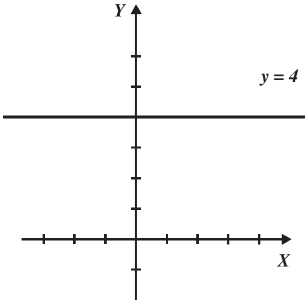
Las demostraciones de los ejercicios 11, 12, 13, 14, 15, 16, 17 y 18, se dejan al estudiante.

EJERCICIO 3

1. $(-\infty, \infty) = \{x \in R\}$
2. $(-\infty, \infty) = \{x \in R\}$
3. $(-\infty, -3) \cup (-3, \infty) = \{x \in R | x \neq -3\}$
4. $(-\infty, 5) \cup (5, \infty) = \{x \in R | x \neq 5\}$
5. $(-\infty, -4) \cup (-4, 4) \cup (4, \infty) = \{x \in R | x \neq -4, x \neq 4\}$
6. $(-\infty, 0) \cup (0, 5) \cup (5, \infty) = \{x \in R | x \neq 0, x \neq 5\}$
7. $(-\infty, 2) \cup (2, 5) \cup (5, \infty) = \{x \in R | x \neq 2, x \neq 5\}$
8. $(-\infty, -5) \cup (-5, 5) \cup (5, \infty) = \{x \in R | x \neq -5, x \neq 5\}$
9. $(-\infty, \infty) = \{x \in R\}$
10. $(-\infty, -5) \cup (-5, 0) \cup (0, \infty) = \{x \in R | x \neq -5, x \neq 0\}$
11. $(-\infty, -1) \cup (-1, 0) \cup (0, 1) \cup (1, \infty) = \{x \in R | x \neq -1, x \neq 0, x \neq 1\}$
12. $[-1, \infty) = \{x \geq -1\}$
13. $[6, \infty) = \{x \geq 6\}$
14. $(-\infty, 2] = \{x \leq 2\}$
15. $(-\infty, 4] = \{x \leq 4\}$
16. $(-\infty, -5] \cup [5, \infty) = \{x \in R | x \leq -5 \text{ o } x \geq 5\}$
17. $(-\infty, -1] \cup [6, \infty) = \{x \in R | x \leq -1 \text{ o } x \geq 6\}$
18. $[-6, 6] = \{x \in R | -6 \leq x \leq 6\}$
19. $(-\infty, \infty) = \{x \in R\}$
20. $(-\infty, \infty) = \{x \in R\}$
21. $[5, \infty) = \{x \in R | x \geq 5\}$
22. $(2, \infty) = \{x \in R | x > 2\}$
23. $(-\infty, 3) = \{x \in R | x < 3\}$
24. $(-\infty, -2) \cup (-2, \infty) = \{x \in R | x \neq -2\}$
25. $(-\infty, -4] \cup (3, \infty) = \{x \in R | x \leq -4 \text{ o } x > 3\}$
26. $\left[1, \frac{3}{2}\right) = \left\{x \in R | 1 \leq x < \frac{3}{2}\right\}$
27. $(-2, \infty) = \{x \in R | x \geq -2\}$
28. $\left(-\infty, \frac{5}{2}\right) = \left\{x \in R | x < \frac{5}{2}\right\}$
29. $(0, \infty) = \{x \in R | x > 0\}$
30. $(-1, 3) = \{x \in R | -1 < x < 3\}$
31. $[1, \infty) = \{y \in R | y \geq 1\}$
32. $[-4, \infty) = \{y \in R | y \geq -4\}$
33. $(-\infty, 9] = \{y \in R | y \leq 9\}$
34. $\left(-\infty, \frac{9}{4}\right] = \left\{y \in R | y \leq \frac{9}{4}\right\}$
35. $(-\infty, -2) \cup (-2, \infty) = \{y \in R | y \neq -2\}$
36. $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right) = \left\{y \in R | y \neq \frac{1}{2}\right\}$
37. $[1, \infty) = \{y \in R | y \geq 1\}$
38. $(-\infty, 0] = \{y \in R | y \leq 0\}$
39. $[0, 2] = \{y \in R | 0 \leq y \leq 2\}$
40. $(0, 1] = \{y \in R | 0 < y \leq 1\}$
41. $[0, 1) \cup (1, \infty) = \{x \in R | 0 \leq y < 1 \text{ o } y > 1\}$
42. $[0, \infty) = \{y \in R | y \geq 0\}$

Ejercicio 4

1.



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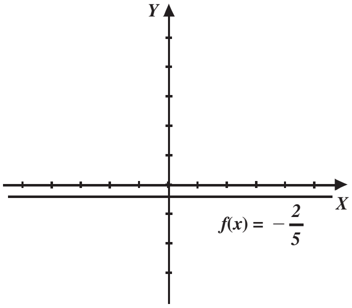

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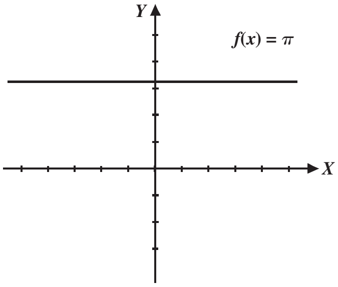

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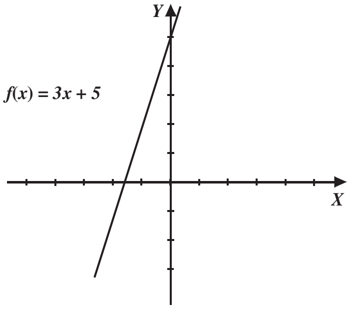

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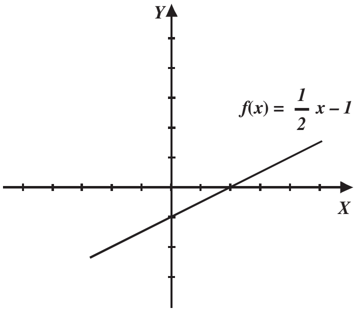

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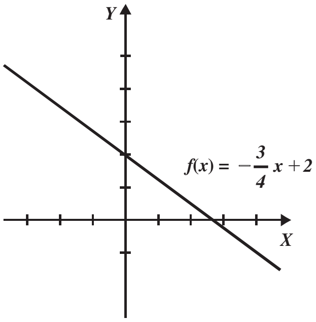

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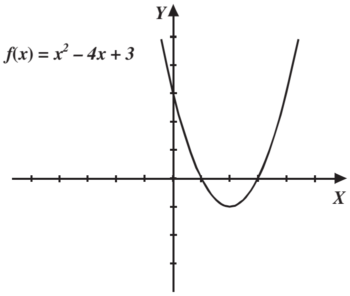

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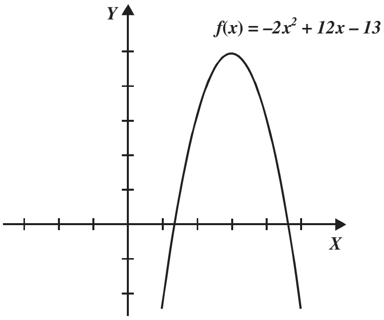

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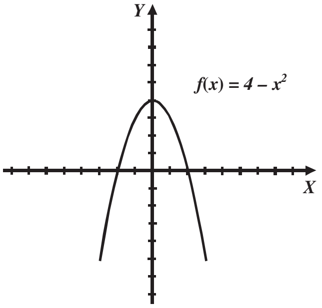

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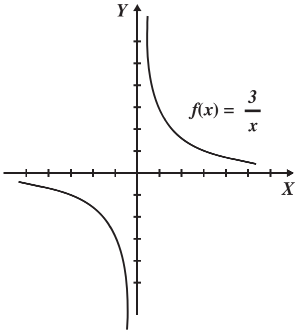

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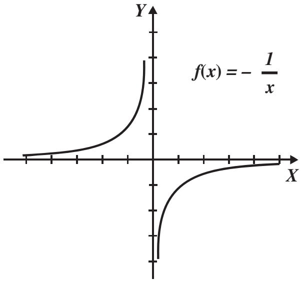

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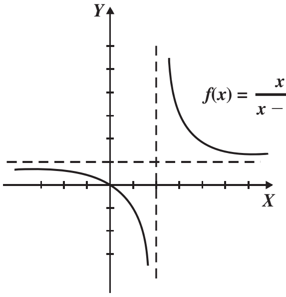

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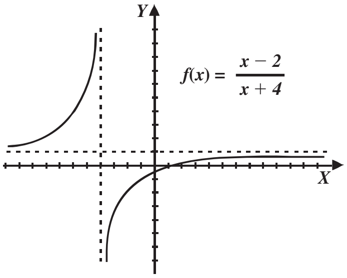

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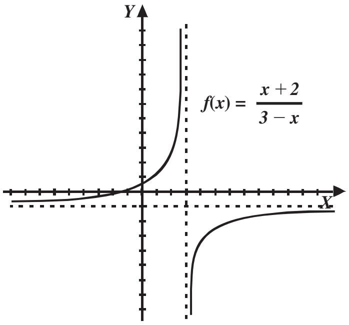

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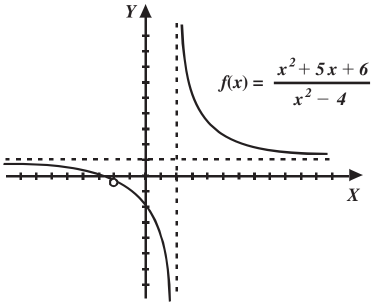

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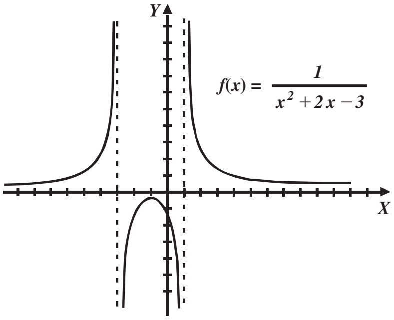

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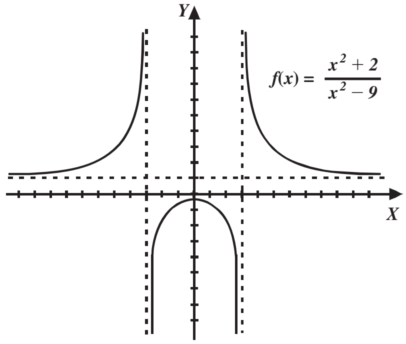

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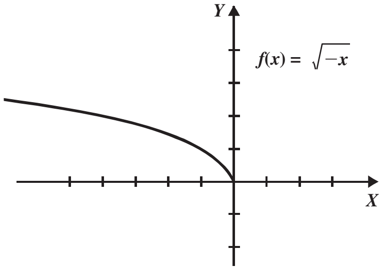

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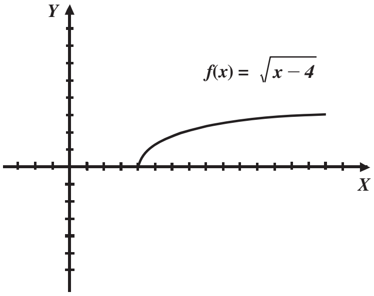

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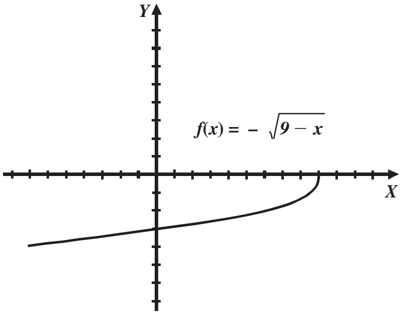

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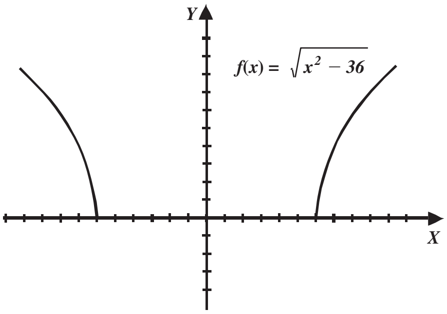

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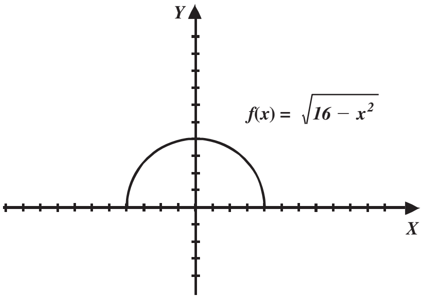

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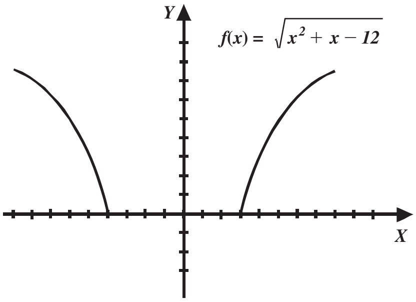

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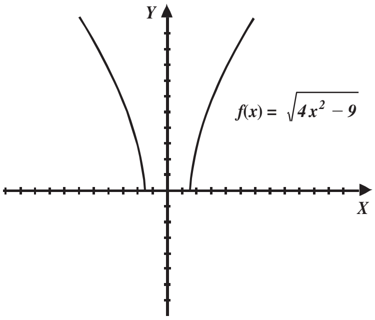

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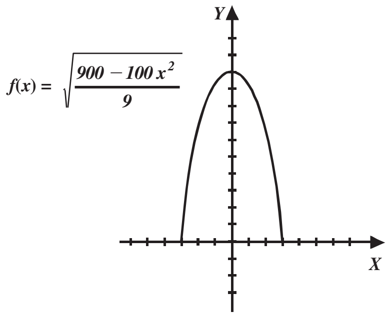

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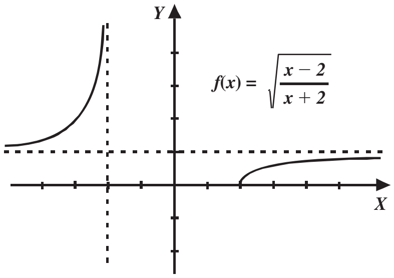

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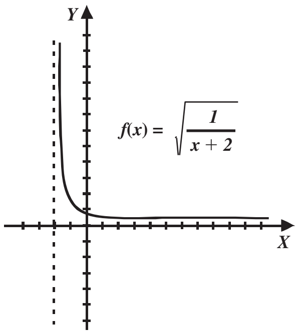

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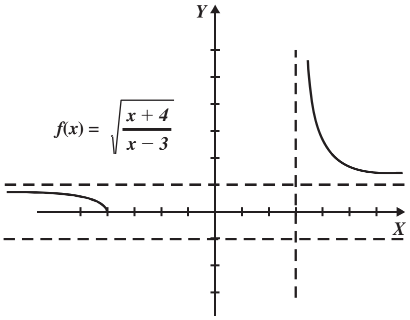

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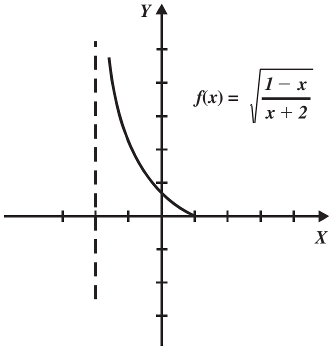

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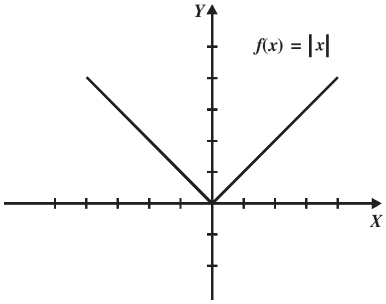

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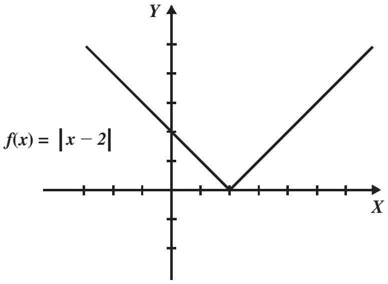

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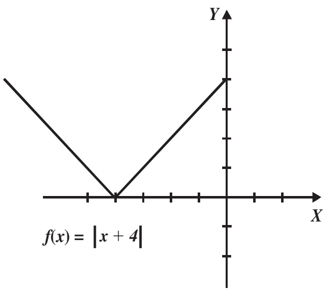

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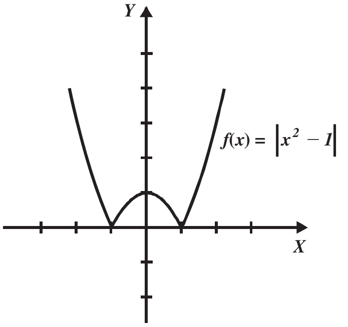

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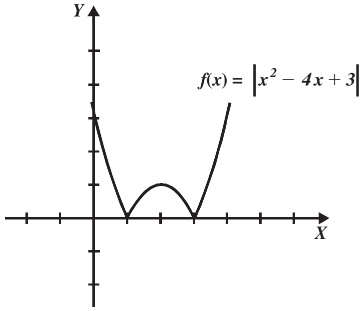

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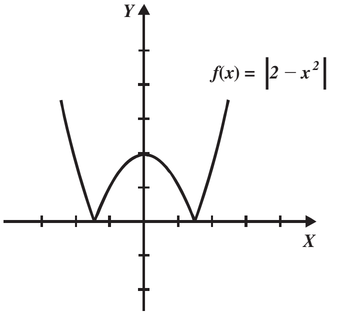

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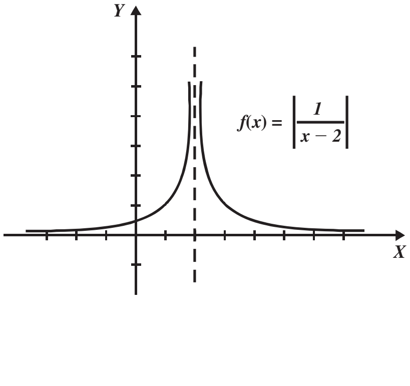

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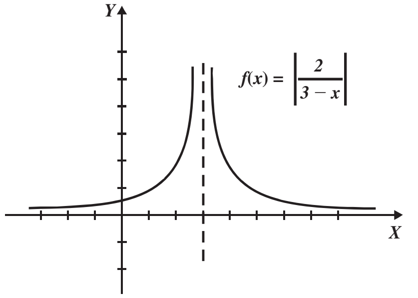

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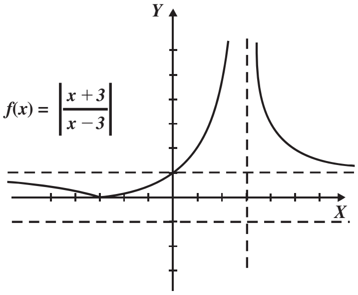

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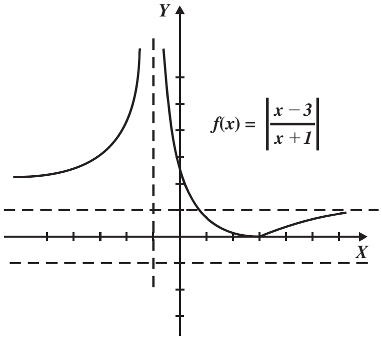

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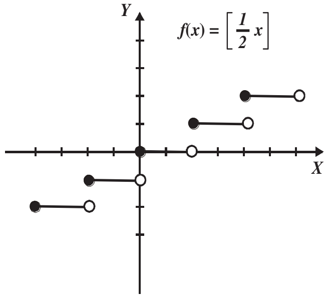

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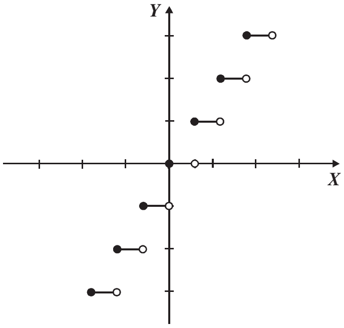

EJERCICIO 5

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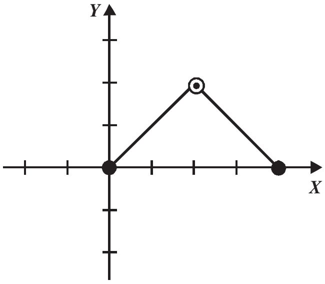

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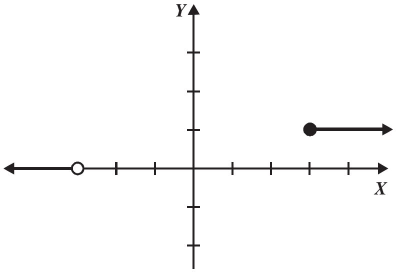

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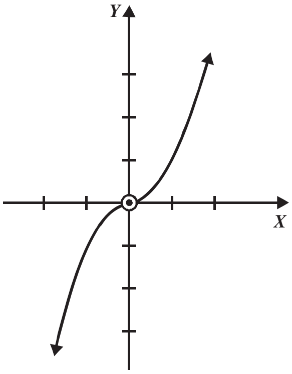

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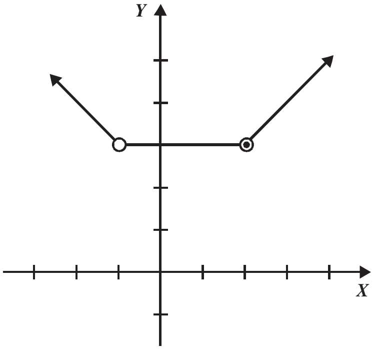

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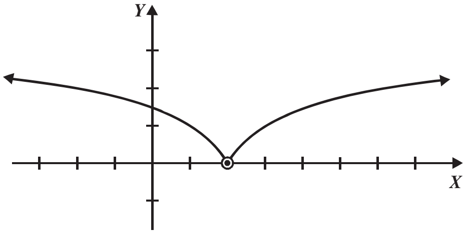

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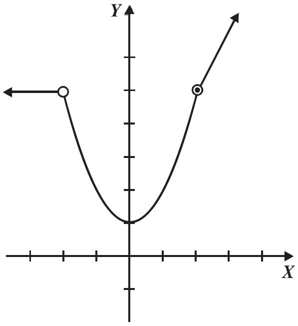

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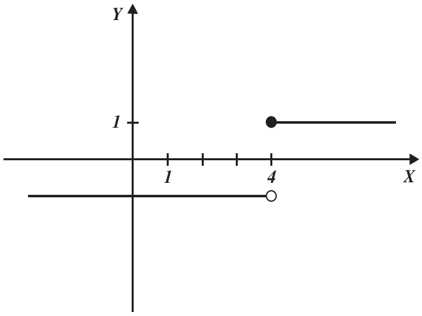

Ejercicio 6

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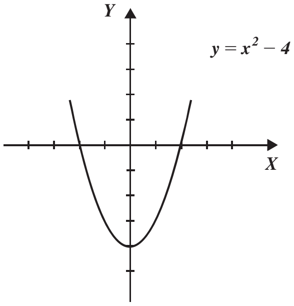

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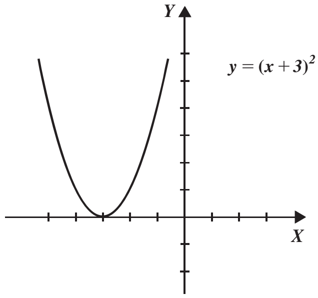

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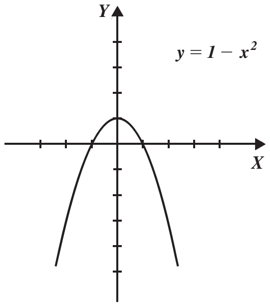

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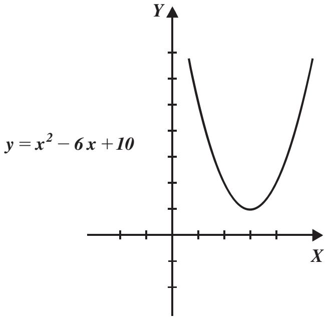

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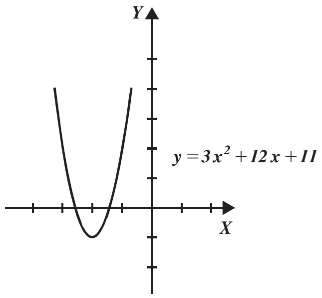

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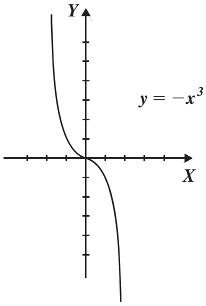

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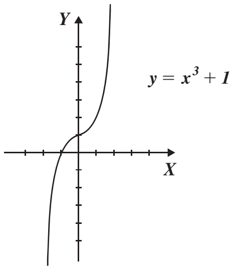

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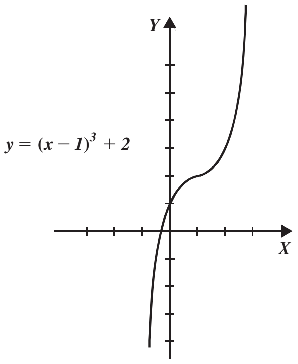

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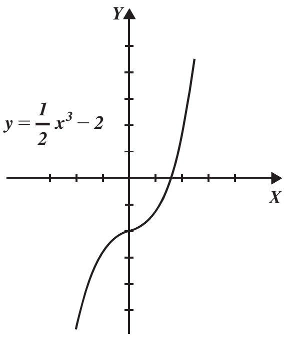

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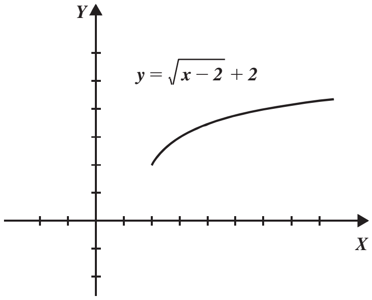

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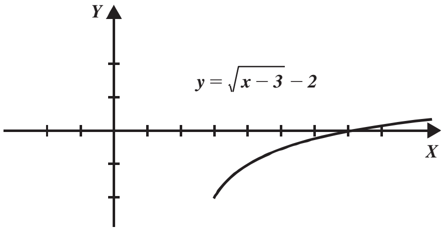

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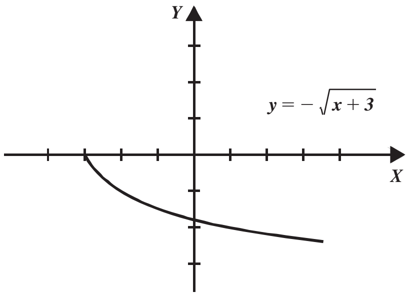

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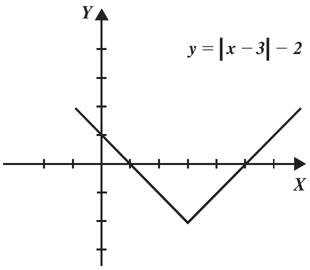
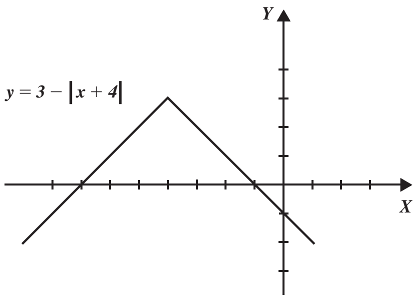


14.



EJERCICIO 7

1. Crece: $(0, \infty)$
2. Decrece: $(-\infty, 0)$
   Crece: $(0, \infty)$
3. Crece: $(-\infty, +\infty)$
4. Decrece: $(-\infty, 0)$
   Crece: $(0, \infty)$
5. Crece: $(2, \infty)$
6. Decrece: $(-\infty, -3)$
7. Decrece: $(0, \infty)$
   Crece: $(-\infty, 0)$
8. Decrece: $(-\infty, 3)$
   Crece: $(3, \infty)$
9. Decrece: $(0, 3)$
   Crece: $(-3, 0)$
10. No crece ni decrece, permanece constante

EJERCICIO 8

1. Biyectiva
2. Ninguna
3. Ninguna
4. Biyectiva
5. Inyectiva
6. Ninguna
7. Biyectiva
8. Inyectiva
9. Suprayectiva
10. Biyectiva

EJERCICIO 9

1. $f(x) + g(x) = 3$
   $f(x) - g(x) = 7$
   $f(x) \cdot g(x) = -10$
   $\frac{f(x)}{g(x)} = -\frac{5}{2}$
2. $f(x) + g(x) = 4x$
   $f(x) - g(x) = -10$
   $f(x) \cdot g(x) = 4x^2 - 25$
   $\frac{f(x)}{g(x)} = \frac{2x-5}{2x+5}$

3. $f(x) + g(x) = 2x^2 - x - 3$
$f(x) - g(x) = -7(x + 1)$
$f(x) \cdot g(x) = x^4 - x^3 - 15x^2 - 23x - 10$
$\dfrac{f(x)}{g(x)} = \dfrac{x-5}{x+2}$

4. $f(x) + g(x) = \dfrac{8x+1}{6}$
$f(x) - g(x) = \dfrac{4x-7}{6}$
$f(x) \cdot g(x) = \dfrac{2x^2+3x-2}{6}$
$\dfrac{f(x)}{g(x)} = \dfrac{6x-3}{2x+4}$

5. $f(x) + g(x) = \sqrt{x-3} + \sqrt{x+4}$
$f(x) - g(x) = \sqrt{x-3} - \sqrt{x+4}$
$f(x) \cdot g(x) = \sqrt{x^2+x-12}$
$\dfrac{f(x)}{g(x)} = \sqrt{\dfrac{x-3}{x+4}} = \dfrac{\sqrt{x^2+x-12}}{x+4}$

6. $f(x) + g(x) = x + 2\sqrt{x}$
$f(x) - g(x) = x$
$f(x) \cdot g(x) = x + x\sqrt{x}$
$\dfrac{f(x)}{g(x)} = \sqrt{x} + 1$

7. $f(x) + g(x) = \text{sen}^2 x + \cos^2 x = 1$
$f(x) - g(x) = \text{sen}^2 x - \cos^2 x = -\cos 2x$
$f(x) \cdot g(x) = \text{sen}^2 x \cdot \cos^2 x = \dfrac{1}{4}\,\text{sen}^2 2x$
$\dfrac{f(x)}{g(x)} = \dfrac{\text{sen}^2 x}{\cos^2 x} = \tan^2 x$

8. $D_f \cap D_g = \{-1, 3, 5\}$
$f + g = \{(-1, 12), (3, 20), (5, 23)\}$
$f - g = \{(-1, -8), (3, -8), (5, -9)\}$
$f \cdot g = \{(-1, 20), (3, 84), (5, 112)\}$
$\dfrac{f}{g} = \left\{\left(-1, \dfrac{1}{5}\right), \left(3, \dfrac{3}{7}\right), \left(5, \dfrac{7}{16}\right)\right\}$

9. $D_f \cap Dg = \{-2, -1, 0\}$
$f + g = \{(-2, 0), (-1, 1), (0, 2)\}$
$f - g = \{(-2, -10), (-1, -7), (0, -4)\}$
$f \cdot g = \{(-2, -25), (-1, -12), (0, -3)\}$
$\dfrac{f}{g} = \left\{(-2, -1), \left(-1, -\dfrac{3}{4}\right), \left(0, -\dfrac{1}{3}\right)\right\}$

10. $D_f \cap D_g = \{-1, 1, 2\}$
$f + g = \{(-1, 1), (1, 1), (2, 1)\}$
$f - g = \{(-1, -3), (1, 1), (2, 0)\}$
$f \cdot g = \left\{(-1, -2), (1, 0), \left(2, \dfrac{1}{4}\right)\right\}$
$\dfrac{f}{g} = \left\{\left(-1, -\dfrac{1}{2}\right), (2, 1)\right\}$

11. $f(x) + r(x) = 2x + 5$
12. $f(x) - s(x) = -x^2 + 4x + 13$
13. $g(x) \cdot s(x) = x^4 + 2x^3 - 19x^2 - 68x - 60$
14. $\dfrac{g(x)}{r(x)} = x + 3$
15. $\dfrac{s(x)}{r(x)} = x - 5$
16. $g(x) - s(x) = 8(x + 2)$
17. $f(x) \cdot r(x) = x^2 + 5x + 6$
18. $\dfrac{f(x)}{r(x)} = \dfrac{x+3}{x+2}$
19. $\dfrac{g(x)}{s(x)} = \dfrac{x+3}{x-5}$
20. $\dfrac{g(x)}{f(x)} + \dfrac{s(x)}{r(x)} = 2x - 3$
21. $f(x) + g(x) = \dfrac{x^2+2}{x(x+2)}$
22. $\dfrac{f(x)}{g(x)} = \dfrac{x^2-x}{x+2}$
23. $f(x) \cdot g(x) = \dfrac{x-1}{x^2+2x}$
24. $f(x) - h(x) = \dfrac{5-5x}{(x+2)(x-3)}$
25. $g(x) \cdot h(x) = \dfrac{x-1}{x^2-3x}$
26. $\dfrac{f(x)}{g(x)} + h(x) = \dfrac{x^3-3x^2+4x-2}{(x+2)(x-3)}$
27. $\dfrac{h(x)}{f(x)} - g(x) = \dfrac{x^2+x+3}{x(x-3)}$
28. $\dfrac{h(2) - f(1)}{g(3)} = -3$
29. $f(x + 1) \cdot \dfrac{1}{h(x+1)} = \dfrac{x-2}{x+3}$
30. $h(x) - g(x) = \dfrac{x^2-2x+3}{x(x-3)}$

31. $\dfrac{h(x)}{g(x)} - \dfrac{g(x)}{f(x)} = \dfrac{x^4 - 2x^3 + x + 6}{x(x-1)(x-3)}$

32. $f(x) \cdot h(x) - g(x) = \dfrac{x^3 - 3x^2 + 2x + 6}{x(x+2)(x-3)}$

33. $\dfrac{f(x) + h(x)}{g(x)} = \dfrac{x(x-1)(2x-1)}{(x+2)(x-3)}$

34. $\dfrac{1}{g(x) + h(x)} = \dfrac{x(x-3)}{x^2 - 3}$

35. $\dfrac{1}{1 - h(x)} = \dfrac{3 - x}{2}$

EJERCICIO 10

1. $(f \circ g)(x) = 12x^2 - 46x + 40$, $(g \circ f)(x) = 6x^2 - 10x - 7$,
   $(f \circ f)(x) = 27x^4 - 90x^3 + 24x^2 + 85x + 20$,
   $(g \circ g)(x) = 4x - 9$
2. $(f \circ g)(x) = x$, $(g \circ f)(x) = x$, $(f \circ f)(x) = \sqrt[4]{x}$, $(g \circ g)(x) = x^4$
3. $(f \circ g)(x) = 4$, $(g \circ f)(x) = 2$, $(f \circ f)(x) = 4$, $(g \circ g)(x) = 2$
4. $(f \circ g)(x) = x$, $(g \circ f)(x) = x$, $(f \circ f)(x) = \sqrt{x^2 - 10}$
   $(g \circ g)(x) = \sqrt{x^2 + 10}$
5. $(f \circ g)(x) = x + 2\sqrt{x-1}$, $(g \circ f)(x) = \sqrt{x^2 + 2x}$
   $(f \circ f)(x) = (x^2 + 2x + 2)^2$, $(g \circ g)(x) = \sqrt{\sqrt{x-1} - 1}$
6. $(f \circ g)(x) = \dfrac{1 - x}{1 + 3x}$, $(g \circ f)(x) = \dfrac{x + 3}{x - 1}$
   $(f \circ f)(x) = -\dfrac{1}{2x + 1}$, $(g \circ g)(x) = x$
7. $(f \circ g)(x) = \log(x - 4)$, $(g \circ f)(x) = \log(x - 2) - 2$
   $(f \circ f)(x) = \log[\log(x - 2) - 2]$, $(g \circ g)(x) = x - 4$
8. $(f \circ g)(x) = \dfrac{1}{\sqrt{x}}$, $(g \circ f)(x) = \sqrt{-x^2 - \sqrt{x^4 - 1}}$
   $(f \circ f)(x) = \sqrt{-\dfrac{1}{x^2}}$ no está definida
   $(g \circ g)(x) = \sqrt{x + \sqrt{x^2 - 1}}$
9. $(f \circ g)(x) = \{(1, 5), (2, 6), (3, 7), (4, 8)\}$
   $(g \circ f)(x) =$ No está definida, $(f \circ f)(x) = \{(2, 8)\}$
   $(g \circ g)(x) = \{(1, 3), (2, 4), (3, 5)\}$
10. $(f \circ g)(x) = \{(-2, 1), (-1, 4), (0, 9), (1, 16)\}$
    $(g \circ f)(x) = (1, 4)$, $(f \circ f)(x) = \{(1, 1), (2, 16)\}$
    $(g \circ g)(x) = \{(-2, 4)\}$
11. $(f \circ g)(x) = \{(3, 1), (-2, -3), (1, -1)\}$
    $(g \circ f)(x) = \{(0, -1), (1, 0)\}$, $(f \circ f)(x) = \{(0, 3), (-1, -1)\}$
    $(g \circ g)(x) = \{(-2, -2)\}$
12. $f(x) = \dfrac{1}{x}$
13. $f(x) = \sqrt{x}$
14. $f(x) = mx + b$
15. $f(x) = x^2$
16. $f(x) = \sqrt{x - 2}$
17. $f \circ g \circ h = 81x^2 - 54x + 9$
18. $f \circ g \circ h = 1 - 12x^2 + 48x^4 + 64x^6$
19. $f \circ g \circ h = \sqrt{2x - 9}$
20. $f \circ g \circ h = \text{sen}^2(x - 2)$
21. $f \circ g \circ h = \sec^2 x$
22. $f \circ g \circ h = \text{sen } x$

EJERCICIO 11

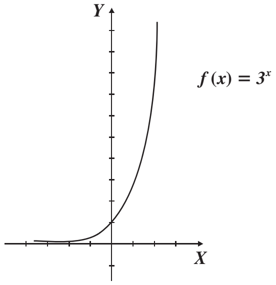
1. Ninguna	6. Ninguna	11. Par
2. Par	7. Par	12. Par
3. Impar	8. Par	13. Impar
4. Ninguna	9. Ninguna	14. Par
5. Ninguna	10. Impar	15. Par

EJERCICIO 12

1. $f^{-1}(x) = x$
2. $f^{-1}(x) = \dfrac{x + 5}{2}$
3. $f^{-1}(x) = \sqrt{x + 9}$
4. No tiene inversa
5. $f^{-1}(x) = \sqrt[3]{x}$
6. $f^{-1}(x) = \sqrt[5]{x}$
7. $f^{-1}(x) = \sqrt[4]{x}$
8. $f^{-1}(x) = 3 - x^2$
9. No tiene inversa
10. $f^{-1}(x) = \sqrt{4 - x^2}$
11. $f^{-1}(x) = x^3 - 9$
12. $f^{-1}(x) = \dfrac{1 - 3x}{2x}$
13. $f^{-1}(x) = \sqrt{x^2 + 1}$
14. $f^{-1}(x) = \dfrac{1 + x}{1 - x}$
15. $f^{-1}(x) = \dfrac{x^2}{x^2 - 1}$

Ejercicio 13

1.



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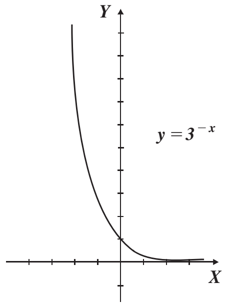

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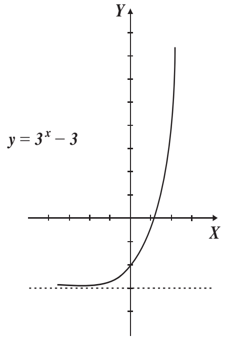

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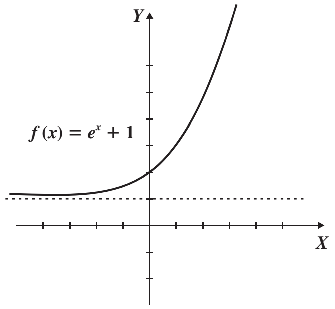

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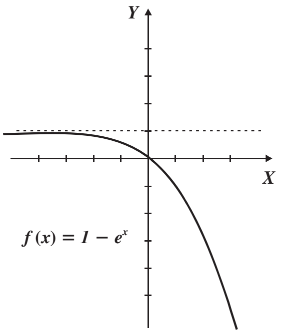

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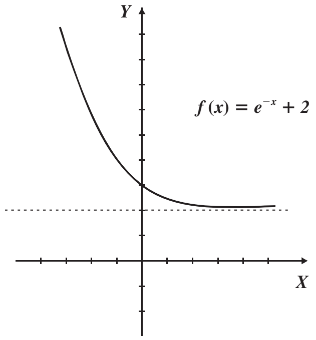

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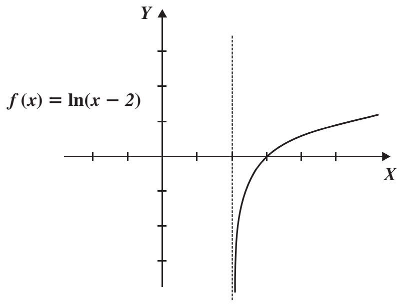

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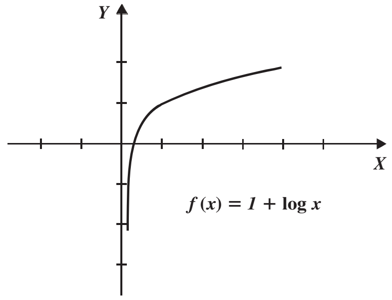

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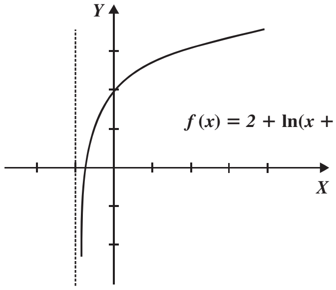

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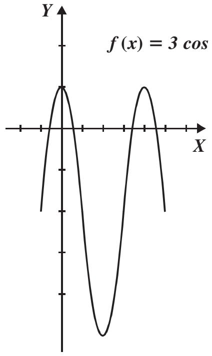

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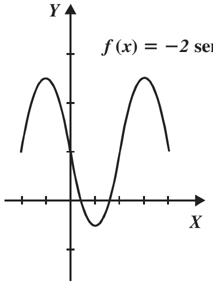

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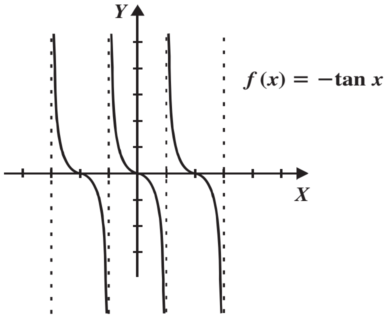

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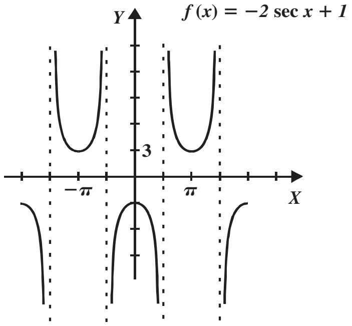
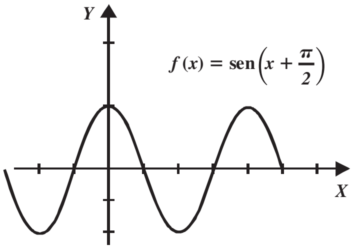


14.



EJERCICIO 14

1. $V(h) = 40h$
2. $V(h) = \frac{4}{75}\pi h^3$
3. $P(A) = \frac{12\sqrt{5A}}{5}$
4. $A(d) = \frac{\pi d^2}{4}$
5. $V(x) = \frac{\sqrt{3}}{2}\pi x^3$
6. $A(x) = \frac{3}{4}(x + 2)^2$
7. $V(x) = \frac{\pi r^2}{6}(8r + 15)$
8. $A(x) = \frac{\pi x^2}{3}$
9. $A(x) = 3x\sqrt{16 - x^2}$
10. $A(x) = (x - 4)\left(\frac{540}{x} - 3\right)$
11. $d(t) = \frac{9}{2}\sqrt{16t^2 + 1}$
12. $d(t) = \frac{1}{2}\sqrt{t^2 - 16}$

Capítulo 2

Ejercicio 15

1. -1
2. $0.16666 = \frac{1}{6}$
3. -1
4. 0
5. 1
6. 4
7. No existe
8. No existe
9. 2
10. 3

Ejercicio 16

11. $\delta = 0.01$
12. $\delta = 0.08$
13. $\delta = 0.025$
14. $\delta = 0.4$
15. $\varepsilon = 0.18$
16. $\varepsilon = 0.0098$
17. $\varepsilon = 0.25$
18. $\varepsilon = 0.002$

Ejercicio 17

1. 3
2. 24
3. 18
4. 0
5. 7
6. 64
7. -3
8. $-\frac{4}{27}$
9. 15
10. $2\sqrt{3}$
11. 32
12. 1
13. 1
14. $-\frac{5}{8}$
15. 0
16. No existe
17. 1
18. $\frac{\sqrt{2}}{4}$
19. 1
20. h
21. $\frac{4\sqrt{3}}{3}$

Ejercicio 18

1. $\frac{3}{5}$
2. No existe
3. 0
4. $\frac{a}{c}$
5. $-\frac{2}{3}$
6. $\frac{1}{2}$
7. 4
8. $\frac{1}{2h}$
9. $-\frac{1}{2}$
10. $2a$
11. 9
12. 0
13. $-\frac{1}{2}$
14. $\frac{5}{4}$
15. $\frac{1}{4}$
16. 2
17. 4
18. $-\frac{3}{7}$
19. 15
20. $\frac{9}{19}$
21. $\frac{7}{9}$
22. -6
23. $\frac{1}{3}$
24. -3
25. 3
26. $\frac{1}{12}$
27. 48
28. $\frac{1}{4}$
29. 2
30. No existe
31. $\frac{1}{2}$
32. $2\sqrt{5}$
33. $-\frac{1}{6}$
34. $\frac{24}{5}$
35. $\frac{b}{a}$
36. $\frac{1}{n \cdot \sqrt[n]{p^{n-1}}}$
37. $-\frac{1}{2}$
38. $\frac{5}{12}$
39. 2
40. 0
41. $-\frac{3}{20}$
42. $\frac{1}{4}$
43. $\frac{1}{4}$
44. $\frac{1}{6a\sqrt[3]{a^2}}$
45. $\frac{1}{3}$
46. $\frac{1}{8}$
47. $\frac{1}{2\sqrt[4]{x}}$
48. $\frac{1}{54}$
49. $\frac{1}{4}$
50. $\frac{2}{25}$

Ejercicio 19

1. $\frac{7}{4}$
2. 2
3. 0
4. No existe
5. 3
6. $\frac{1}{2}$
7. $-\frac{1}{3}$
8. 0
9. 1
10. 0
11. $-\frac{11}{6}$
12. -1
13. $\frac{9}{2}$
14. 1
15. 1
16. $\frac{a_m}{b_n}$ si $m = n$

    0 si $m < n$

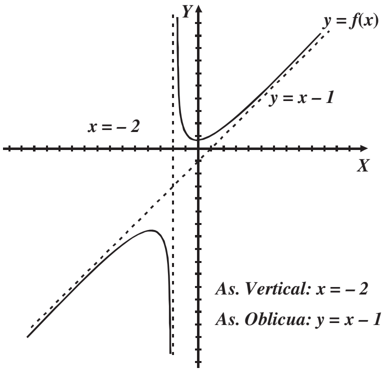
    No existe si $m > n$
17. $\frac{a}{c}$
18. $\sqrt[n]{a}$

EJERCICIO 20

1. $y = \dfrac{1}{2}$
2. $y = 0$
3. No tiene asíntota horizontal
4. $y = 1, y = -1$
5. $y = 2$
6. $y = \dfrac{a}{c}$
7. $y = 0$
8. $y = -2$
9. No tiene asíntota horizontal
10. $y = \dfrac{a}{b}$

EJERCICIO 21

1.



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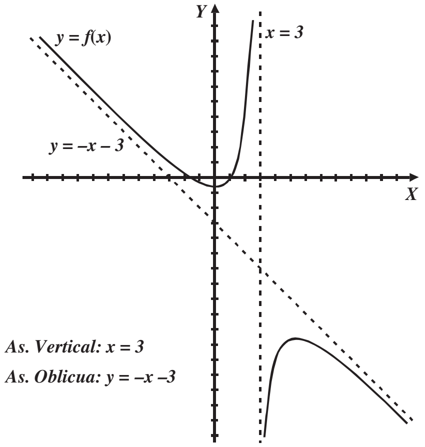

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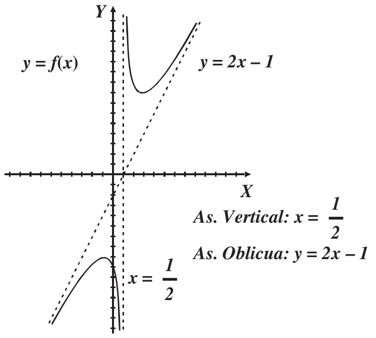

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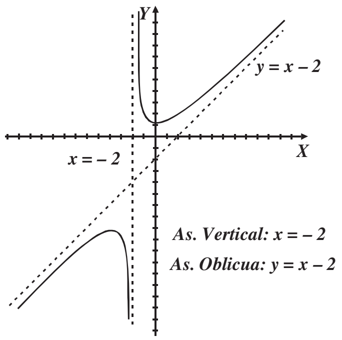

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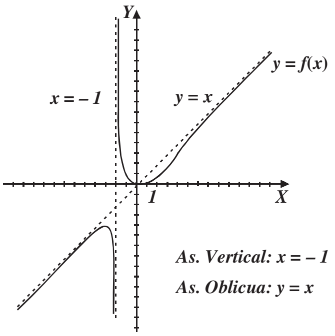

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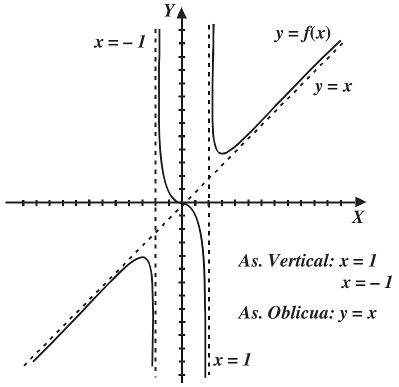

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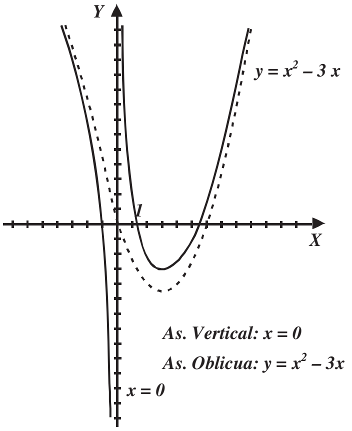
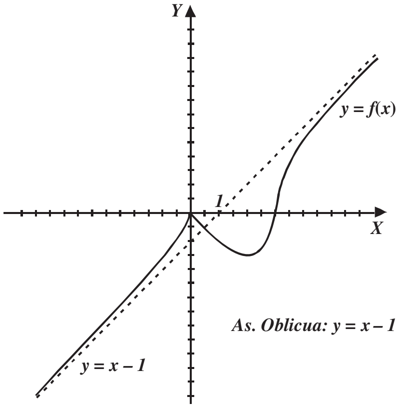


8.



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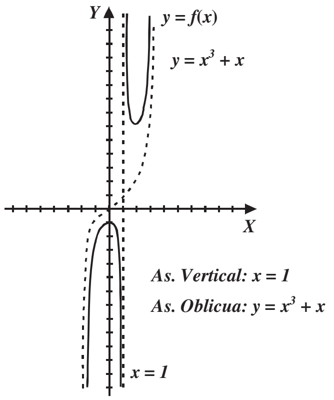


EJERCICIO 22

1. *a*) 11, *b*) 9, *c*) No existe
2. *a*) -1, *b*) -1, *c*) -1, *d*) -6, *e*) -4, *f*) No existe
3. *a*) 0, *b*) 2, *c*) No existe, *d*) $-\frac{2}{3}$, *e*) $-\frac{2}{3}$, *f*) $-\frac{2}{3}$
4. *a*) 1, *b*) 4, *c*) 4, *d*) 4, *e*) 16
5. *a*) 4, *b*) 4, *c*) 4, *d*) 8, *e*) 3, *f*) No existe
6. -7
7. 1
8. 3
9. 2
10. No existe el límite

EJERCICIO 23

1. $\frac{1}{3}$
2. $\frac{1+\sqrt{3}}{2}$
3. -2
4. -1
5. $-\frac{1}{2}$
6. 2
7. $-4\sqrt{3}$
8. 0
9. 1
10. No existe

EJERCICIO 24

1. -2
2. $\frac{3}{4}$
3. $-\frac{1}{2}$
4. 0
5. $-\frac{1}{2}$
6. 0
7. 0
8. $\sqrt{2}$
9. -1
10. $-\sec^3(3)$
11. -2
12. $\frac{1}{2^m}$
13. 1
14. 0
15. $\frac{1}{2}$
16. 0
17. 9
18. 0
19. $\frac{n^2-m^2}{4}$
20. 0

CAPÍTULO 3

EJERCICIO 25

1. Es continua en $x = 0$
2. No es continua en $x = 2$
3. No es continua en $x = -\frac{3}{2}$
4. Es continua en $x = 3$
5. Continuidad removible en $x = 2$
6. No es continua en $x = 2\pi$
7. Es continua en $x = 2$
8. No es continua en $x = 1$
9. No es continua en $x = 0$

10. No es continua en $x = -2$; es continua en $x = 2$
11. Es continua en $x = 1$; no es continua en $x = 2$
12. Es continua en $x = \pi$ y $x = \frac{3}{2}\pi$
13. No es continua en $x = -3$; es continua en $x = 3$
14. Continuidad removible en $x = 3$
15. Continuidad removible en $x = 1$
16. Continuidad removible en $x = -2$
17. Continuidad removible en $x = 8$
18. No es continua en $x = \frac{1}{2}$
19. $k = 1$
20. $k = 0$ o $k = 2$
21. $k = 1$ o $k = -\frac{2}{9}$
22. $a = -\frac{9}{4}, b = -7$
23. $a = -\frac{17}{2}, b = -2$
24. $a = 4, b = 2$

EJERCICIO 26

1. sí
2. no
3. sí
4. no
5. sí
6. sí
7. sí
8. no
9. no
10. sí

EJERCICIO 27

1. 2
2. 0
3. 4
4. $\pm\sqrt{2}, \frac{1}{2}$
5. 5
6. -5
7. 5
8. $\frac{2}{5}$
9. $\frac{\sqrt{2}}{2}$
10. 1

CAPÍTULO 4

EJERCICIO 28

1. $y' = 3$
2. $y' = -b$
3. $y' = 2x$
4. $f'(x) = 6x - 5$
5. $y' = 2ax + b$
6. $y' = 3x^2$
7. $y' = 3x^2 - 2x$
8. $y' = 4$
9. $y' = -\frac{2}{(x-1)^2}$
10. $y' = 3x^2$
11. $f'(x) = -\frac{6}{x^3}$
12. $f'(x) = \frac{4x}{(x^2+1)^2}$
13. $f'(x) = \frac{1}{2\sqrt{x-2}}$
14. $f'(x) = \frac{x}{\sqrt{x^2-4}}$
15. $y' = \frac{2}{3\sqrt[3]{(2x+1)^2}}$
16. $y' = -\frac{1}{x\sqrt{x}}$
17. $y' = \frac{1}{3\sqrt[3]{x^2}}$
18. $y' = \frac{-2}{3(x-1)\sqrt[3]{x-1}}$
19. $y' = \frac{2}{(x-1)^{\frac{1}{2}}(x+3)^{\frac{3}{2}}}$
20. $y' = \frac{1}{n\sqrt[n]{x^{n-1}}}$

EJERCICIO 29

1. $y' = 0$
2. $y' = 0$
3. $f'(x) = 0$
4. $s'(t) = 0$
5. $y' = 6$
6. $y' = \frac{3}{4}$
7. $f'(x) = a$
8. $s'(t) = b^2$
9. $f'(x) = 5\sqrt{2}$
10. $y' = a\sqrt{b}$
11. $f'(x) = 5x^4$
12. $f'(x) = 12x^2$
13. $s'(t) = \frac{4}{5}t^3$
14. $y' = \frac{9}{2}x^{\frac{7}{2}}$
15. $f'(x) = \frac{4}{3}x^{\frac{1}{3}}$
16. $y' = 9x^{\frac{1}{2}}$
17. $f'(x) = \frac{2}{5\sqrt[5]{x^3}}$
18. $f'(x) = \frac{1}{\sqrt[4]{x^3}}$
19. $f'(x) = \frac{1}{2\sqrt{x}}$
20. $s'(t) = \frac{1}{4\sqrt[4]{t^3}}$
21. $f'(x) = \frac{1}{\sqrt[5]{x^4}}$
22. $f'(x) = \frac{5x^4}{7}$
23. $f'(x) = \frac{4x^3}{9}$
24. $s'(t) = \frac{3t^2}{a}$
25. $f'(x) = -\frac{20}{x^5}$
26. $f'(x) = -\frac{12}{x^7}$
27. $f'(x) = \frac{1}{4\sqrt{x}}$
28. $s'(t) = \frac{1}{15\sqrt[3]{t^2}}$
29. $f'(x) = -\frac{2}{x\sqrt{x}}$
30. $s'(t) = -\frac{5}{4t\sqrt[4]{t}}$

31. $f'(x) = -\dfrac{4}{3x\sqrt[3]{x}}$

32. $f'(x) = 21x^2 - 6x + 3$

33. $f'(x) = 4x^3 - 15x^2 + 16x - 1$

34. $f'(x) = 10x + 4$

35. $f'(x) = 12ax^3 - 12ax^2 - 10bx + 7c$

36. $f'(x) = \dfrac{x^2}{2} - \dfrac{6x}{5} - \dfrac{4}{9}$

37. $s'(t) = \dfrac{5t^4}{6} - \dfrac{4t^3}{5} + \dfrac{3t^2}{4} - \dfrac{2t}{7} + \dfrac{1}{9}$

38. $f'(x) = \dfrac{2x}{\sqrt{a^2 + b^2}} - \dfrac{1}{a}$

39. $s'(t) = -\dfrac{8}{t^3} + \dfrac{5}{t^2}$

40. $f'(x) = -\dfrac{20}{x^5} + \dfrac{18}{x^4} + \dfrac{14}{x^3} + \dfrac{3}{x^2}$

41. $s(t) = \dfrac{3t^2}{5} + \dfrac{4}{t^3} - \dfrac{6}{t^2}$

42. $f'(x) = \dfrac{1}{15\sqrt[3]{x^2}} + \dfrac{1}{x\sqrt[3]{x}}$

43. $f'(x) = 3x^2 - 6x - 6 - \dfrac{2}{x^2}$

44. $f'(x) = 3\sqrt{x} + \dfrac{5}{4\sqrt{x}} + \dfrac{2}{x^2\sqrt{x}}$

45. $f'(x) = \dfrac{4}{\sqrt{x}} + \dfrac{6}{\sqrt[3]{x}} + 6\sqrt{x}$

46. $f'(x) = anx^{n-1} + b(n-1)x^{n-2}$

47. $f'(x) = \dfrac{2}{3}x + \dfrac{5}{7}$

48. $f'(x) = \dfrac{a}{n\sqrt[n]{x^{n-1}}} + \dfrac{b}{3\sqrt[3]{x^2}}$

49. $y' = \dfrac{1}{3\sqrt[3]{x}} + \dfrac{5\sqrt[4]{x}}{12}$

50. $f'(x) = -\dfrac{5}{2x^2\sqrt[4]{x}} + \dfrac{1}{2x\sqrt{x}} + 3$

51. $f'(x) = 14x + 15x^2$

52. $f'(x) = -\dfrac{6}{x^3} - \dfrac{5}{x^2} - 2$

53. $f'(x) = 5\sqrt[3]{x^2} + \dfrac{10}{3\sqrt[3]{x}} - \dfrac{8}{3x\sqrt[3]{x}}$

54. $y' = \dfrac{3}{x^2}$

55. $y' = 15(3x - 4)^4$

56. $y' = -12(2 - 4x)^2$

57. $y' = (72x^5 - 32x^3)(3x^6 - 2x^4)^3$

58. $y' = 12\sqrt{x}\,(2x - 1)^2(6x - 1)$

59. $y' = \dfrac{-3x}{\sqrt{5 - 3x^2}}$

60. $y' = \dfrac{x^2}{\sqrt[3]{(x^3 + 2)^2}}$

61. $y' = \dfrac{1 - x^2}{(x^2 + 1)^2}$

62. $y' = \dfrac{4x + 6}{3\sqrt{2x^2 + 6x}}$

63. $y' = \left(1 + \dfrac{9}{\sqrt{x}}\right)\left(\dfrac{x}{3} + 6\sqrt{x}\right)^2$

64. $f'(x) = \dfrac{x^3}{\sqrt[4]{(x^4 - 2)^3}}$

65. $f'(x) = (6x + 15)(x^2 + 5x - 3)^2$

66. $y' = \dfrac{4}{3\sqrt[3]{2x - 3}}$

67. $y' = \dfrac{1}{(4x + 3)^{\frac{3}{4}}}$

68. $f'(x) = \left(\dfrac{1}{3}x + 2\right)^2$

69. $y' = \left(\dfrac{2}{x} - \dfrac{1}{x^2}\right)^{-\frac{1}{2}}\left(-\dfrac{1}{x^2} + \dfrac{1}{x^3}\right)$

70. $f'(z) = \dfrac{z}{\sqrt{z^2 - 4}}$

71. $y' = \dfrac{2x^5 + 1}{\sqrt[3]{(x^6 + 3x)^2}}$

72. $y' = 108x^2 + 55x - 4$

73. $y' = 40x - 12 - \dfrac{9}{x^2}$

74. $y' = 12x^3 + 3x^2$

75. $f'(x) = \dfrac{3x + 1}{\sqrt{2x + 1}}$

76. $y' = \frac{1}{3}(2x+1)^2\left(8x+1\right)$

77. $y' = \frac{5x^2 - 4x}{2\sqrt{x-1}}$

78. $f'(x) = 12x(3x^2 - 5)^3(2x^2 + 1)^2(7x^2 - 3)$

79. $f'(\theta) = (6\theta^4 - 12\theta)(\theta^2 + 1)^2(2\theta^3 + \theta - 2)$

80. $s' = \frac{8 - 9t}{2\sqrt{4 - 3t}}$

81. $s'(t) = (2t + 3)\left(\frac{2}{t} - \frac{3}{t^2}\right) = 4 - \frac{9}{t^2}$

82. $f'(x) = \frac{6}{(1-2x)^2}$

83. $f'(t) = \frac{-bt}{a\sqrt{a^2 - t^2}}$

84. $f'(r) = \frac{r^3 - 5r}{(r^2 - 4)^{\frac{3}{2}}}$

85. $f'(t) = \frac{63}{(5t+8)^2}$

86. $f'(z) = \frac{21}{(5-6z)^2}$

87. $f'(x) = -\frac{2ab}{(ax-b)^2}$

88. $f'(x) = -\frac{1}{x\sqrt{x}} - \frac{1}{2\sqrt{3x}}$

89. $f'(t) = \frac{-2}{\left(1+2t\right)\sqrt{1-4t^2}}$

90. $f'(w) = \frac{10(w-3)}{(w+2)^3}$

91. $f'(\theta) = \frac{24\theta^3 - 54\theta^2 + 24}{(3-2\theta)^2}$

92. $f'(s) = \frac{-6s^2 + 4s - 12}{(s^2 - 6s)^2}$

93. $f'(x) = \frac{10b^2x + 5x^3}{2(b^2 + x^2)^{\frac{3}{2}}}$

94. $f'(t)\ \frac{(693 - 27t)(9t-6)^2}{(27 - 3t)^3}$

95. $f'(x) = \frac{2ab}{(a-3x)^2}$

96. $f'(x) = \frac{8 - 4x^2}{\sqrt{4 - x^2}}$

97. $y' = \frac{-4x^3}{(x^4 - a^4)^{\frac{3}{2}}}$

98. $y' = \frac{7x^4 + 27x^2}{3(x^2+3)^{\frac{4}{3}}}$

99. $y' = \frac{8x^2 + 24x + 9}{2\sqrt{x^2 + 3x}}$

100. $y' = \frac{x+2}{2(x+1)^{\frac{3}{2}}}$

101. $y' = \frac{3x - 3}{2\sqrt{x-3}}$

102. $y' = \frac{-2x^2}{(x^6 - 1)^{\frac{2}{3}}(x^3 - 1)^{\frac{2}{3}}}$

103. $y' = \frac{nx^{n-1}}{(1 - x^n)\sqrt{x^{2n} - 1}}$

104. $y' = \frac{-n\sqrt[m]{x^{n-m}}}{m\left(\sqrt[m]{x^n} - 1\right)^2}$

105. $y' = \frac{-20x^2 + 19x + 8}{2(4-5x)^{\frac{3}{2}}\sqrt{2x+1}}$

106. $y' = \frac{8x^6 - 8x^3 - 2}{\sqrt[3]{(4x^6 - 1)^2(2x^3 - 1)^2}}$

EJERCICIO 30

1. $\frac{dy}{dx} = \frac{1-2u}{x^2}$

2. $\frac{dy}{dx} = \frac{1}{2\sqrt{x}\sqrt{u^2 - 1}(1+u)}$

3. $\frac{dy}{dx} = \frac{x(6u^2 - 3)}{\sqrt{2u^3 - 3u}}$

4. $\frac{dy}{dx} = \frac{4}{u^3} - \frac{9}{u^4}$

5. $\frac{dy}{dx} = \frac{(8 + 12x - 3x^2)(u^2+1)}{(u^2-1)^2}$

6. $\frac{dy}{dx} = \frac{5u^3 + 3u}{\sqrt{u^3 + u}}$

7. $\frac{dy}{dx} = \frac{3x^2}{\sqrt{u}(x^3+1)^2}$

8. $\frac{dy}{dx} = \frac{8x}{(u-1)^2(v-2)^2\sqrt{x^2 - 1}}$

9. $\frac{dy}{dx} = -\frac{v}{2\sqrt{x}\sqrt{u-1}(v^2-1)^2}$

10. $\frac{dy}{dx} = -\frac{2x(x^2+3)}{(v+1)\sqrt{v^2-1}\sqrt{u^3}}$

11. $\frac{dy}{dx} = -\frac{2u}{\sqrt{v}(x-1)^2}$

EJERCICIO 31

1. $y' = 8\cos 8x$
2. $f'(x) = -6x \operatorname{sen} 3x^2$
3. $f'(x) = 3x^2 \sec^2 x^3$
4. $s'(t) = 6\sec 6t \tan 6t$
5. $f'(x) = -12x^2 \csc^2 4x^3$
6. $f'(x) = -9\csc 9x \cot 9x$
7. $f'(x) = -a \operatorname{sen} ax$
8. $s'(t) = 2bt \sec^2 bt^2$
9. $f'(x) = 12x \sec x^2 \tan x^2$
10. $f'(x) = -\frac{1}{8}\csc\frac{x}{4}\cot\frac{x}{4}$
11. $f'(x) = -3a \operatorname{sen} 3x$
12. $f'(x) = -3\csc^2(3x-5)$
13. $f'(x) = \cos\frac{x}{2}$
14. $f'(x) = -5\operatorname{sen}\left(5x - \frac{\pi}{2}\right)$
15. $s'(t) = a\sec^2(at+\pi)$
16. $f'(x) = \cos x - \operatorname{sen} x$
17. $s'(t) = \frac{1}{2\sqrt{t}}\cos\sqrt{t}$
18. $f'(x) = -\frac{1}{3\sqrt[3]{x^2}}\csc^2\sqrt[3]{x}$
19. $f'(x) = -\frac{1}{x^2}\cos\frac{1}{x}$
20. $s'(t) = \frac{3}{t^4}\operatorname{sen}\frac{1}{t^3}$
21. $f'(x) = -\frac{\sec\frac{1}{\sqrt{x}}\tan\frac{1}{\sqrt{x}}}{2x\sqrt{x}}$
22. $f'(x) = 3\sec^2 3x - 3 = 3\tan^2 3x$
23. $f'(x) = a - a\csc^2 ax = -a\cot^2 ax$
24. $f'(x) = 2(x-1)\cos(x-1)^2$
25. $f'(x) = -18t(3t^2+2)^2 \operatorname{sen}(3t^2+2)^3$
26. $f'(x) = -\frac{2}{\sqrt{x-1}}\csc^2\sqrt{x-1}$
27. $f'(x) = -\frac{2}{(x-1)^2}\sec^2\left(\frac{x+1}{x-1}\right)$
28. $f'(x) = -\frac{2ab}{(ax-b)^2}\sec\left(\frac{ax+b}{ax-b}\right)\tan\left(\frac{ax+b}{ax-b}\right)$
29. $f'(x) = 10\operatorname{sen} 5x\cos 5x = 5\operatorname{sen} 10x$
30. $f'(x) = -3b\cos^2 bx \operatorname{sen} bx$
31. $f'(x) = 24x\tan^3 3x^2 \sec^2 3x^2$
32. $f'(x) = \frac{2\cos 4x}{\sqrt{\operatorname{sen} 4x}} = 2\sqrt{\operatorname{sen} 4x}\cot 4x$
33. $f'(x) = 5x\tan 5x^2\sqrt{\sec 5x^2}$
34. $f'(x) = \frac{2x\sec^2 x^2}{\sqrt[3]{9\tan^2 x^2}}$
35. $f'(x) = x\cos x + \operatorname{sen} x$
36. $f'(x) = 2x\cos x^2 - 2x^3\operatorname{sen} x^2$
37. $f'(x) = \frac{3x\cos 3x - \operatorname{sen} 3x}{x^2}$
38. $f'(x) = -\frac{10t^2\operatorname{sen} 5t^2 + 2\cos 5t^2}{t^3}$
39. $y' = 2ax\cos(ax^2)$
40. $y' = -3a\operatorname{sen}(3x)$
41. $y' = \frac{\sec^2\sqrt{x}}{2\sqrt{x}}$
42. $y' = x\sec 3x^2\tan 3x^2$
43. $y' = -\frac{1}{3}\csc\frac{2x}{3}\cot\frac{2x}{3}$
44. $y' = 2x + 3 + \frac{1}{x^2}\cos\left(\frac{1}{x}\right)$
45. $y' = -6x\csc^2(1-x^2)$
46. $y' = \frac{-4}{3(x-1)^2}\cdot\cos\left(\frac{x+1}{x-1}\right)$
47. $y' = 2b\operatorname{sen} 4bx$
48. $y' = 24(2x-1)^2\tan^3(2x-1)^3\sec^2(2x-1)^3$
49. $y' = \frac{\operatorname{sen} 2x}{\sqrt{\cos^3 2x}}$
50. $y' = \frac{2x\sec^2 x^2}{\sqrt[3]{9\tan^2 x^2}}$
51. $y' = \cos 4x(\cos^2 4x - 6x\operatorname{sen} 8x)$
52. $y' = x\csc ax(2 - ax\cot ax)$
53. $y' = (1 - x\cot 2x)\sqrt{\csc 2x}$

54. $y' = \dfrac{-m \operatorname{sen} nx \operatorname{sen} mx - n \cos nx \cos mx}{\operatorname{sen}^2 nx}$

55. $y' = \dfrac{-\cos x}{(1 + \operatorname{sen} x)^2}$

56. $y' = -x \operatorname{sen} x$

57. $y' = \dfrac{\sec^2 x (\tan x - 1)}{\sqrt{(\tan^2 x - 1)^3}}$

58. $y' = (2x^2 - 6) \cos 2x + 10x \operatorname{sen} 2x$

59. $y' = -2 \cos(2x - 1)$

60. $y' = x \sec(\pi - x) \cdot [2 - x \tan(\pi - x)]$

61. $y' = \dfrac{81x^2 \operatorname{sen}^2 x [3x^2 \cos x + x \cos x + \operatorname{sen} x]}{(3x+1)^4}$

62. $y' = \dfrac{1}{(x-1)\sqrt{x^2-1}} \operatorname{sen} \sqrt{\dfrac{x+1}{x-1}}$

63. $y' = \left(\dfrac{\sec x}{x}\right)^2 (x \operatorname{sen} x - \cos x)$

64. $y' = -x \operatorname{sen} x + \cos x$

65. $y' = 2 \cos 2x$

66. $y' = 2 \sec^2 x \tan x$

67. $y' = -\dfrac{1}{2} \operatorname{sen} \dfrac{x}{2}$

68. $y' = -6 \operatorname{sen}(6x + 2)$

69. $y' = -\dfrac{x \operatorname{sen} x + 2 \cos x}{x^3}$

70. $y' = (\operatorname{sen} x + \cos x)(\tan^2 x - \tan x + 2)$

71. $y' = -\cos^3 x$

72. $y' = x^2 \operatorname{sen} x$

73. $y' = \operatorname{sen}^4 2x$

EJERCICIO 32

1. $y' = \dfrac{5}{\sqrt{1 - 25x^2}}$

2. $f'(x) = \dfrac{-8x}{\sqrt{1 - 16x^4}}$

3. $f'(x) = \dfrac{3}{1 + 9x^2}$

4. $y' = -\dfrac{3x^2}{1 + x^6}$

5. $f'(x) = \dfrac{2}{x\sqrt{x^4 - 1}}$

6. $f'(x) = -\dfrac{2}{x\sqrt{9x^4 - 1}}$

7. $f'(x) = -\dfrac{1}{\sqrt{b^2 - x^2}}$

8. $f'(x) = \dfrac{1}{\sqrt{16 - x^2}}$

9. $f'(x) = \dfrac{a}{a^2 + x^2}$

10. $f'(x) = \dfrac{1}{x\sqrt{x - 1}}$

11. $y' = -\dfrac{2x}{\sqrt{-8 + 6x^2 - x^4}}$

12. $y' = \dfrac{1}{\sqrt{1 - x^2}}$

13. $y' = \dfrac{x^2}{x^2 + 1} + 2x \operatorname{arc} \tan x$

14. $y' = \operatorname{arc} \operatorname{sen} x$

15. $y' = \dfrac{x^2}{\sqrt{16 - x^2}}$

16. $y' = \operatorname{arc} \csc\left(\dfrac{1}{x}\right) + \sqrt{\dfrac{1+x}{1-x}}$

17. $y' = x \operatorname{arc} \tan x$

18. $\varphi' = \dfrac{\theta}{(1 - \theta^2)\sqrt{\theta^2 - 2}}$

19. $y' = \sqrt{1 - 4x^2}$

20. $y' = x^2 \operatorname{arc} \operatorname{sen} x$

21. $f'(r) = \sqrt{\dfrac{b - r}{b + r}}$

22. $y' = \dfrac{x^2}{x^2 + 1}$

23. $y' = \dfrac{3 + 6x^2}{(1 + 4x^2)(1 + x^2)}$

24. $y' = \dfrac{1}{2\sqrt{x - x^2}}$

25. $y' = \dfrac{1}{\sqrt{x^2 - 1}} + \operatorname{arc} \cos\left(\dfrac{1}{x}\right)$

26. $y' = \dfrac{4a - 8x}{\sqrt{1 - (4ax - 4x^2)^2}}$

27. $f'(r) = \dfrac{1}{\sqrt{-r^2 + 4r - 3}}$

28. $y' = \dfrac{1}{4x^2 + 4x + 5}$

29. $y' = \dfrac{x+2}{\sqrt{4x - x^2}}$

30. $y' = \dfrac{x^2}{\sqrt{4x - x^2}}$

31. $y' = \sqrt{2x - x^2}$

32. $s'(t) = \dfrac{2 - 3t}{\sqrt{9 - t^2}}$

33. $y' = \sqrt{25 - 9x^2}$

34. $w' = \dfrac{\theta + 5}{(\theta + 4)\sqrt{\theta + 2}}$

35. $y' = \dfrac{1}{5 + 4\cos x}$

36. $y' = \dfrac{\cos x}{5 - 3\cos x}$

37. $y' = -\dfrac{1}{3}$

38. $y' = \text{arc cot}(\tan x) - x$

39. $y' = \dfrac{1}{(4x^2 - 1)}\left(\dfrac{1}{x} - \dfrac{4x \text{ arc sec } 2x}{\sqrt{4x^2 - 1}}\right)$

40. $y' = \dfrac{\sqrt{7 - 8\cos x + \cos^2 x}}{14 - 2\cos x}$

41. $y' = \dfrac{64}{(3x + 2)\sqrt{5x^2 + 28x - 12}}$

42. $s' = \dfrac{t^2}{\sqrt{2t - t^2}} + 2t \text{ arc cos}(1 - t) + 2$

43. $y' = -\dfrac{1}{\sqrt{1 - (a + x)^2}}$

Ejercicio 33

1. $y' = \dfrac{3}{x}$

2. $f'(x) = \dfrac{2}{x}$

3. $f'(x) = \dfrac{6x - 5}{3x^2 - 5x + 2}$

4. $f'(x) = \dfrac{1}{2x}$

5. $f'(x) = \dfrac{6 \log e}{x}$

6. $f'(x) = \dfrac{3 \log e}{x}$

7. $f'(x) = \dfrac{\log_3 e}{x}$

8. $f'(x) = \dfrac{\log_4 e}{3x}$

9. $f'(x) = \dfrac{4 \ln^3 x}{x}$

10. $f'(x) = \dfrac{3 \ln^2 5x}{x}$

11. $y' = x(1 + \ln x^2)$

12. $y' = 2(1 + \ln x)$

13. $y' = \dfrac{1 - \ln x}{x^2}$

14. $f'(x) = \dfrac{2 - \ln x^2}{x^2}$

15. $y' = -\dfrac{a}{2(b - ax)} = \dfrac{a}{2(ax - b)}$

16. $f'(x) = \dfrac{9x^2 - 2}{x(3x^2 - 1)}$

17. $f'(x) = \dfrac{2ax^2 - b}{x(ax^2 - b)}$

18. $y' = \dfrac{13}{(3x - 5)(2x + 1)}$

19. $f'(x) = \dfrac{bc}{c^2x^2 - b^2}$

20. $y' = \cot x$

21. $y' = -5 \tan 5x$

22. $y' = \dfrac{2x}{x^2 - 4}$

23. $y' = \dfrac{3}{2(3x + 4)}$

24. $y' = \dfrac{12}{4x^2 - 9}$

25. $y' = \dfrac{x^2}{x^3 + 8}$

26. $y' = \dfrac{\ln x}{2x}$

27. $y' = \dfrac{27x^2+12x+6}{(3x+2)(3x^2+2)}$

28. $y' = \dfrac{2\log_3 e}{4x^2-1}$

29. $y' = \dfrac{\left(30\,bx^2\sqrt{x}-3\right)\log e}{10\,bx^3\sqrt{x}-6x}$

30. $y' = \tan x$

31. $y' = 2\cot x$

32. $y' = 1 + \ln x$

33. $y' = -2\tan 2x$

34. $y' = \dfrac{1}{2x\sqrt{\ln x}}$

35. $y' = \sec x$

36. $y' = -\dfrac{1+\operatorname{sen} 2x}{\cos 2x}$

37. $y' = \dfrac{x+\tan x}{x\tan x}$

38. $y' = 2x^2(1 + \ln x^3)$

39. $y' = \dfrac{3\sec^2\sqrt{x}}{2\sqrt{x}\tan x} = \dfrac{3\sec\sqrt{x}\csc\sqrt{x}}{2\sqrt{x}}$

40. $y' = \dfrac{\log e}{2x}$

41. $y' = 2^{x^2+5x}\cdot\ln 2\cdot(2x+5)$

42. $f'(x) = \dfrac{b^{\sqrt{x}}\ln b}{2\sqrt{x}}$

43. $y' = \dfrac{3^{\ln x}\ln 3}{x}$

44. $y' = (x\cos x + \operatorname{sen} x)\cdot 5^{x\operatorname{sen} x}\cdot\ln 5$

45. $y' = 2^{\ln x}(1 + \ln 2)$

46. $y' = 5^x(\ln 5^x + 1)$

47. $y' = 2xe^{x^2} = 2xy$

48. $y' = (6x-2)e^{3x^2-2x+1} = (6x-2)y$

49. $y' = \dfrac{3xe^{\sqrt{3x^2-1}}}{\sqrt{3x^2-1}} = \dfrac{3xy}{\sqrt{3x^2-1}}$

50. $y' = (x\sec^2 x + \tan x)y$

51. $y' = e^{\frac{2x}{b}} + e^{-\frac{2x}{b}}$

52. $y' = \dfrac{8}{(e^{2x}+e^{-2x})^2}$

53. $f'(x) = 4e^{4x}$

54. $f'(x) = 10xe^{5x^2}$

55. $f'(x) = 3e^{3x-1}$

56. $f'(x) = \dfrac{1}{5}e^{\frac{x}{5}}$

57. $f'(x) = \dfrac{1}{3}\sqrt[3]{e^t}$

58. $f'(x) = \dfrac{1}{4}\sqrt[4]{e^x}$

59. $f'(x) = -\dfrac{2}{x^3}e^{\frac{1}{x^2}}$

60. $f'(x) = \dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$

61. $f'(\theta) = \operatorname{sen} 2\theta\cdot e^{\operatorname{sen}^2\theta}$

62. $f(x) = -2\operatorname{sen} 2x\cdot e^{\cos 2x}$

63. $y' = (\operatorname{sen} x + \cos x)e^{x+e^x\operatorname{sen} x}$

64. $f'(x) = (3\ln 5)5^{3x}$

65. $f'(x) = (2\ln 7)7^{2x}$

66. $f'(x) = (2x\ln 5)5^{x^2}$

67. $y' = 2x^{2x}(1 + \ln x)$

68. $y' = x^{\cos x-1}(\cos x - x\ln x^{\operatorname{sen} x})$

69. $y' = \dfrac{\sqrt[x]{x}}{x^2}(1 - \ln x)$

70. $y' = \dfrac{e^{\arctan x}}{1+x^2} = \dfrac{y}{1+x^2}$

71. $y' = \dfrac{1+2x}{2x}$

72. $y' = 3e^{\ln x^2} = 3x^2$

73. $y' = \dfrac{xe^x}{(x+1)^2}$

74. $y' = \dfrac{e^x(x\ln x+\ln x-1)}{2\ln^2 x}$

75. $y' = \dfrac{-1}{x(\ln x-1)\sqrt{\ln^2 x-1}}$

76. $y' = \dfrac{e^{\operatorname{sen} x}\cos x}{(1-e^{\operatorname{sen} x})\sqrt{e^{2\operatorname{sen} x}-1}}$

77. $y' = \dfrac{2a\cot ax}{e^y}$

78. $y' = \dfrac{e^{\ln\sqrt{e^x\operatorname{sen} x}}(1+\cot x)}{2} = \dfrac{y(1+\cot x)}{2}$

79. $y' = xe^{\operatorname{sen} x}(x\cos x + 2)$

80. $y' = \cot x$

81. $y' = \dfrac{3x^2 - 8}{x(x^2 - 4)}$

82. $y' = \dfrac{x+3}{\sqrt{x^2+9}}$

83. $y' = \sec^3 2x$

84. $y' = \operatorname{arc}\tan x$

85. $y' = \sqrt{x^2 - 4}$

86. $y' = \operatorname{arc}\sec x$

87. $y' = \dfrac{1}{4x^2 - 9}$

88. $y' = \operatorname{arc}\cot x$

89. $y' = \operatorname{arc}\csc \dfrac{x}{2}$

EJERCICIO 34

1. $y' = -\dfrac{x}{y}$

2. $y' = -\dfrac{y}{x}$

3. $y' = \dfrac{4}{y}$

4. $y' = -\dfrac{2x+5}{4y-2}$

5. $y' = -\dfrac{3x+y}{x-6y}$

6. $y' = \dfrac{x+1}{1-y}$

7. $y' = \dfrac{(x-y)^2 + 2y}{2x}$

8. $y' = \dfrac{b^2x}{a^2y}$

9. $y' = -\dfrac{y}{x}$

10. $y' = \dfrac{2y^2 + 3x^2y + 10xy^2}{3y^2 - 4xy - x^3 - 10x^2y + 1}$

11. $y' = \dfrac{4xy - 9x^2 - 5y - 3}{5x - 2x^2 - 1}$

12. $y' = \dfrac{2\sqrt{x+y} - y}{2x + 3y}$

13. $y' = \dfrac{2y\sqrt{x+y} - 1}{1 - 2x\sqrt{x+y}}$

14. $y' = \dfrac{2 - 4x - 3y}{3x + 3}$

15. $y' = \left(1 - 4\sqrt{x}\right)\sqrt{\dfrac{y}{x}}$

16. $y' = \dfrac{y}{x\,(2y-1)}$

17. $y' = -\dfrac{y}{x}$

18. $y' = \dfrac{\tan e^y}{e^y}$

19. $y' = 3e^{x-y}$

20. $y' = \dfrac{2xy}{x^2+1}$

21. $y' = \dfrac{1-x+y}{1-y+x}$

22. $y' = \dfrac{x(1-e^{x^2})}{y(e^{y^2}-1)}$

23. $y' = -x$

24. $y' = -\dfrac{y}{x\ln x}$

25. $y' = \dfrac{y}{x^2 + y^2 + x}$

26. $y' = \dfrac{y}{x}\left(\dfrac{x\ln y - y}{y\ln x - x}\right)$

27. $y' = \dfrac{1}{x\ln x}$

28. $y' = \dfrac{e^x(1+e^y)}{e^y} = (e^{-y} + 1)\ln(1 + e^y)$

29. $y' = \dfrac{y(x - \ln y)}{x\ln x}$

30. $y' = \dfrac{e^x}{1 + x\,e^y} = \dfrac{e^y}{1-y}$

31. $y' = \dfrac{y^2}{e^{\ln y} - xy} = \dfrac{y}{1-x}$

32. $y' = \dfrac{1}{e^{x+y}[\cos(e^{x+y}) - 1]} - 1$

33. $y' = \dfrac{e^{x\cos y}\cos y - 3}{x\,e^{x\cos y}\operatorname{sen} y} = \dfrac{1}{x}\left(\dfrac{1}{\tan y} - \dfrac{1}{x\operatorname{sen} y}\right)$

34. $y' = -\dfrac{\cos(x+a)}{\text{sen}(y-b)}$

35. $y' = \dfrac{\text{sen } x}{\cos y - 1} = -\dfrac{\text{sen } x(\csc y + \cot y)}{\text{sen } y}$

36. $y' = \dfrac{\text{sen}(4x)\cos(2x)}{\text{sen}(2y)\cos(2y)} = \dfrac{\text{sen}(8x)}{\text{sen}(8y)}$

37. $y' = -\dfrac{\text{sen } x\, e^{\cos x}}{\cos y(1 + e^{\text{sen } y})}$

38. $y' = \dfrac{2 - y\cos(xy)}{x\cos(xy)} = \dfrac{2\sec(xy) - y}{x}$

39. $y' = -\dfrac{\cos x}{\text{sen } y}$

40. $y' = \dfrac{\text{sen } x}{\text{sen } y \cdot e^{\cos y}}$

41. $y' = \dfrac{\cos x - \text{sen } y - 1}{x\cos y}$

42. $y' = \left(\dfrac{y^2+1}{y^2+1-x}\right)\text{arc tan } y$

43. $y' = \dfrac{\cot(x+y)}{1-\cot(x+y)}$

44. $y' = \dfrac{1}{2^y \ln 2} = \dfrac{1}{\ln 2^x + \ln 8}$

45. $y' = -\dfrac{y}{e^{\text{sen } y}\cos y + x - 2}$

46. $y' = \dfrac{y}{x}\left(\dfrac{x\ln y - y}{y\ln x - x}\right) = \dfrac{y^2}{x^2}\left(\dfrac{1-\ln x}{1-\ln y}\right)$

47. $y' = \dfrac{\text{sen}(x+y)+\cos(x+y)}{1-\text{sen}(x+y)-\cos(x+y)}$

48. $y' = \dfrac{\tan^2(xy) + y^2\sec^2(xy)}{\tan(xy) - xy\sec^2(xy)}$

49. $y' = \dfrac{x^2+y+1}{(x^2+1)\text{arc cot } x}$

50. $y' = \dfrac{y\cdot e^x}{\sqrt{1-e^{2x}}(\text{sen } y + \text{arc cos}(e^x))}$

EJERCICIO 35

1. $\dfrac{d^4y}{dx^4} = 24$

2. $\dfrac{d^3y}{dx^3} = 0$

3. $\dfrac{d^2y}{dx^2} = -\dfrac{170}{(5x+3)^3}$

4. $\dfrac{d^2y}{dx^2} = \dfrac{4ab^2}{(ax-b)^2}$

5. $\dfrac{d^3y}{dx^3} = 24a^3(ax+b)$

6. $\dfrac{d^4y}{dx^4} = \text{sen } x + \cos x$

7. $y'' = -\csc^2 x$

8. $y''' = -\dfrac{72}{(x-1)^5}$

9. $y'' = e^x\sec^2 e^x(2e^x\tan e^x + 1) = e^x\sec^2 e^x(2e^x y + 1)$

10. $\dfrac{d^2y}{dx^2} = \dfrac{18y-6}{(2-3x)^2}$

11. $\dfrac{d^2y}{dx^2} = -\dfrac{9}{(9-x^2)^{\frac{3}{2}}}$

12. $\dfrac{d^2y}{dx^2} = -\dfrac{x^2+y^2}{y^3} = -\dfrac{16}{y^3}$

13. $\dfrac{d^4y}{dx^4} = \dfrac{2}{x^3}$

14. $\dfrac{d^2y}{dx^2} = -\dfrac{\text{sen } x\,\text{sen}^2 y + \cos^2 x\cos y}{\text{sen}^3 y} = -\csc y\left(\text{sen} x + \dfrac{\cos^2 x\cos y}{\text{sen}^2 y}\right)$

15. $\dfrac{d^3y}{dx^3} = -x^2\cos x - 6x\,\text{sen } x + 6\cos x$

16. $\dfrac{d^3y}{dx^3} = \dfrac{12}{(x+1)^4}$; $\dfrac{d^ny}{dx^n} = \dfrac{2\cdot n!\cdot(-1)^{n+1}}{(x+1)^{n+1}}$

17. $y'' = \dfrac{2y}{(x+1)^2}$

18. $\dfrac{d^3y}{dx^3} = -2\sec^2 x\tan x$

19. $\dfrac{d^2y}{dx^2} = \dfrac{2\cos^2 x}{(1+\text{sen } x)^3} + \dfrac{\text{sen } x}{(1+\text{sen } x)^2} = \dfrac{2-\text{sen } x}{(1+\text{sen } x)^2}$

20. $y'' = \dfrac{-12}{(x+2y)^3}$; $y''' = \dfrac{-108x}{(x+2y)^5}$

Ejercicio 36

1. $\frac{dy}{dx} = \frac{2\ \text{sen}\ 2\theta + 3\cos 2\theta}{2\cos 2\theta - 3\ \text{sen}\ 2\theta}$
2. $\frac{dy}{dx} = 0$
3. $\frac{dy}{dx} = \frac{5\ \text{sen}\ \theta\cos 5\theta + \text{sen}\ 5\theta\cos\theta}{5\cos\theta\cos 5\theta - \text{sen}\ 5\theta\ \text{sen}\ \theta}$
4. $\frac{dy}{dx} = \frac{\cos 2\theta\ \text{sen}\ \theta + \text{sen}\ 2\theta\cos\theta}{\cos 2\theta\cos\theta - \text{sen}\ 2\theta\ \text{sen}\ \theta}$
5. $\frac{dy}{dx} = -\frac{\cos 2\theta + \cos\theta}{\text{sen}\ 2\theta + \text{sen}\ \theta} = -\cot\left(\frac{3\theta}{2}\right)$
6. $\frac{dy}{dx} = \csc\theta - \cot\theta$
7. $\frac{dy}{dx} = \frac{a\ \text{sen}\ \theta + \cos\theta}{a\cos\theta - \text{sen}\ \theta}$
8. $\frac{dy}{dx} = \frac{\text{sen}\ \theta\tan\frac{\theta}{2} + \cos\theta}{\cos\theta\tan\frac{\theta}{2} - \text{sen}\ \theta}$
9. $\frac{dy}{dx} = \frac{3\cos\theta - 2\cos 2\theta}{2\ \text{sen}\ 2\theta - 3\ \text{sen}\ \theta}$
10. $\frac{dy}{dx} = \frac{\text{sen}\ \theta + 2\theta\cos\theta}{\cos\theta - 2\theta\ \text{sen}\ \theta}$
11. -1
12. $2 + \sqrt{3}$
13. 3
14. $-a$
15. $-\frac{1}{4}$

Ejercicio 37

1. $\frac{dy}{dx} = 4t\sqrt{t}$
2. $\frac{dy}{dx} = \frac{1}{36t^2}$
3. $\frac{dy}{dx} = \frac{2\sqrt{t^2 - t}}{2t - 1}$
4. $\frac{dy}{dx} = \frac{a}{b}\csc\theta$
5. $\frac{dy}{dx} = \frac{2t^2 - 3t^3}{2\sqrt{t-1}}$
6. $\frac{dy}{dx} = \frac{\cos\theta + 4\ \text{sen}\ \theta}{5\cos\theta + \text{sen}\ \theta}$
7. $\frac{dy}{dx} = \frac{(t-1)^2(1-2t-t^2)}{(t^2+1)}$
8. $\frac{dy}{dx} = \csc\frac{\theta}{2}(4 - 8\cos^2\theta)$
9. $\frac{dy}{dx} = -\frac{4}{5t^4}$
10. $\frac{dy}{dx} = -\frac{1}{2}\cos\theta\cot^2\theta$
11. $\sqrt{3}$
12. $\frac{1}{4m^2}$
13. $-\frac{1}{3}$
14. $-\frac{1}{3}$
15. 1
16. $-\frac{1}{10}$

Capítulo 5

Ejercicio 38

1. Sub-tangente $= 1$
   Sub-normal $= 1$
   Tangente $= \sqrt{2}$
   Normal $= -\sqrt{2}$
2. Sub-tangente $= -\frac{12}{7}$
   Sub-normal $= -84$
   Tangente $= -\frac{60}{7}\sqrt{2}$
   Normal $= -60\sqrt{2}$
3. Sub-tangente $= -\frac{12}{7}$
   Sub-normal $= -84$
   Tangente $= -\frac{60}{7}\sqrt{2}$
   Normal $= 60\sqrt{2}$
4. Sub-tangente $= \frac{7}{4}$
   Sub-normal $= 28$
   Tangente $= \frac{7\sqrt{17}}{4}$
   Normal $= 7\sqrt{17}$
5. Sub-tangente $= -\frac{3}{2}$
   Sub-normal $= -6$
   Tangente $= -\frac{3}{2}\sqrt{5}$
   Normal $= 3\sqrt{5}$
6. Sub-tangente $= -18$
   Sub-normal $= -\frac{1}{2}$
   Tangente $= -3\sqrt{37}$
   Normal $= \frac{\sqrt{37}}{2}$

7. Sub-tangente = 8

   Sub-normal = $\frac{1}{2}$

   Tangente = $2\sqrt{17}$

   Normal = $\frac{\sqrt{17}}{2}$

8. Sub-tangente = -2

   Sub-normal = $-\frac{1}{8}$

   Tangente = $-\frac{\sqrt{17}}{2}$

   Normal = $\frac{\sqrt{17}}{8}$

9. Sub-tangente = $-\frac{3}{2}$

   Sub-normal = -6

   Tangente = $-\frac{3\sqrt{5}}{2}$

   Normal = $-3\sqrt{5}$

10. Sub-tangente = $-\frac{3}{4}$

    Sub-normal= $-\frac{16}{27}$

    Tangente = $-\frac{\sqrt{145}}{12}$

    Normal = $\frac{2\sqrt{145}}{27}$

11. $T: 4x + y - 1 = 0$
    $N: x - 4y + 38 = 0$
12. $T: x + y - 1 = 0$
    $N: x - y + 1 = 0$
13. $T: y = 0$
    $N: 2x - 1 = 0$
14. $T: 8x - y - 12 = 0$
    $N: x + 8y - 34 = 0$
15. $T: 4x + y + 8 = 0$
    $N: x - 4y - 15 = 0$
16. $T: \sqrt{5}x + 2y - 9 = 0$
    $N: 2x - \sqrt{5}y = 0$
17. $T: x + 2y - 7 = 0$
    $N: 2x - y - 4 = 0$
18. $T: 2x + y - 2 = 0$
    $N: x - 2y + 4 = 0$
19. $T: y - 1 = 0$
    $N: 2x - \pi = 0$
20. $T: 3\sqrt{3}x + 6y - \left(3 + \sqrt{3}\pi\right) = 0$
    $N: 12x - 6\sqrt{3}y + \left(3\sqrt{3} - 4\pi\right) = 0$
21. $T: 4x - 2y + (6 - \pi) = 0$
    $N: 4x + 8y - (24 + \pi) = 0$
22. $T: 2x - y - 6 = 0$
    $N: x + 2y - 8 = 0$
23. $T: x + y - 2 = 0$
    $N: x - y = 0$
24. $T: 6x + 2y - 9 = 0$
    $N: 2x - 6y + 7 = 0$
25. $T: x - 2 = 0$
    $N: 2y - 1 = 0$
26. $T: 7x - y - 8 = 0$
    $N: x + 7y - 94 = 0$
27. $T: x - ey = 0$
    $N: ex + y - 1 - e^2 = 0$
28. $T: 8x + y - 7 = 0$
    $N: 2x - 16y + 47 = 0$

EJERCICIO 39

1. Agudo 26° 33′, obtuso 153° 27′
2. Agudo 73° 42′, obtuso 106° 18′
3. Agudo 78° 41′, obtuso 101° 19′
4. Agudo 35° 15′, obtuso 144° 44′
5. Agudo 28° 23′, obtuso 151° 36′
6. Agudo 28° 4′, obtuso 151° 55′
7. Agudo 71° 33′, obtuso 104° 28′
8. 125° 32′
9. $\theta = 63° 26'$
10. $\theta = 18° 26'$
11. $\theta = 6° 54', 57° 25'$
12. $\theta = 33° 41'$
13. $\theta = 54° 44'$

EJERCICIO 40

1. $r = \frac{5\sqrt{5}}{3}, \frac{d\theta}{ds} = \frac{3\sqrt{5}}{25}$
2. $r = \frac{17\sqrt{17}}{8}, \frac{d\theta}{ds} = \frac{8\sqrt{17}}{289}$
3. $r = 4\sqrt{2}, \frac{d\theta}{ds} = \frac{\sqrt{2}}{8}$
4. $r = \frac{5\sqrt{10}}{3}, \frac{d\theta}{ds} = \frac{3\sqrt{10}}{50}$
5. $r = 1, \frac{d\theta}{ds} = 1$

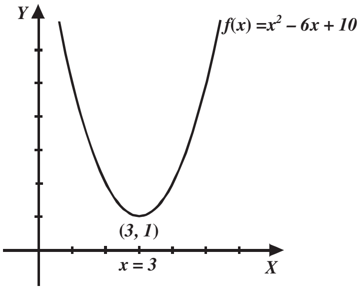
6. $r = \frac{17\sqrt{17}}{2}, \frac{d\theta}{ds} = \frac{2\sqrt{17}}{289}$
7. $r = \frac{1}{2}, \frac{d\theta}{ds} = 2$
8. $r = \frac{\sqrt{11}}{4}, \frac{d\theta}{ds} = \frac{4\sqrt{11}}{11}$
9. $C(-2, 5)$
10. $C\left(-\frac{3}{4}, -\frac{3}{2}\right)$
11. $C\left(\frac{\pi}{2}, 0\right)$
12. $C(-2, 3)$
13. $C\left(\frac{23}{2}, -32\right)$

EJERCICIO 41

1. Punto mínimo $(3, -4)$
   Creciente en $(3, \infty)$
   Decreciente en $(-\infty, 3)$
2. Punto máximo $\left(\frac{5}{6}, -\frac{23}{12}\right)$
   Creciente en $\left(-\infty, \frac{5}{6}\right)$
   Decreciente en $\left(\frac{5}{6}, \infty\right)$
3. Punto máximo $(-1, 2)$
   Punto mínimo $(1, -2)$
   Creciente en $(-\infty, -1) \cup (1, \infty)$
   Decreciente en $(-1, 1)$
4. Punto máximo $(0, 0)$
   Punto mínimo $(4, -32)$
   Creciente en $(-\infty, 0) \cup (4, \infty)$
   Decreciente en $(0, 4)$
5. Punto máximo $(-1, 5)$
   Punto mínimo $\left(\frac{1}{2}, -\frac{7}{4}\right)$
   Creciente en $(-\infty, -1) \cup \left(\frac{1}{2}, \infty\right)$
   Decreciente en $\left(-1, \frac{1}{2}\right)$
6. Punto máximo $\left(-\frac{1}{2}, \frac{17}{4}\right)$
   Punto mínimo $\left(\frac{2}{3}, \frac{29}{27}\right)$
   Creciente en $\left(-\infty, -\frac{1}{2}\right) \cup \left(\frac{2}{3}, \infty\right)$
   Decreciente en $\left(-\frac{1}{2}, \frac{2}{3}\right)$
7. Punto máximo $(2, 15)$
   Punto mínimo $(-1, -12)$
   Creciente en $(-1, 2)$
   Decreciente en $(-\infty, -1) \cup (2, \infty)$
8. Punto máximo $\left(-1, \frac{8}{3}\right)$
   Punto mínimo $(3, -8)$
   Creciente en $(-\infty, -1) \cup (3, \infty)$
   Decreciente en $(-1, 3)$
9. Punto máximo $\left(-2, \frac{34}{3}\right)$
   Punto mínimo $\left(3, -\frac{19}{2}\right)$
   Creciente en $(-\infty, -2) \cup (3, \infty)$
   Decreciente en $(-2, 3)$
10. Punto máximo $\left(1, \frac{17}{12}\right)$
    Punto mínimo $(0, 1)\left(3, -\frac{5}{4}\right)$
    Creciente en $(0, 1) \cup (3, \infty)$
    Decreciente en $(-\infty, 0) \cup (1, 3)$
11. Punto máximo $(1, -3)$
    Creciente $(-\infty, 0) \cup (0, 1)$
    Decreciente $(1, 2) \cup (2, \infty)$
12. No tiene máximos y mínimos
    Decreciente en $(-\infty, 3) \cup (3, \infty)$
13. Punto máximo $\left(2, \frac{1}{2}\right)$
    Punto mínimo $\left(-2, -\frac{1}{2}\right)$
    Creciente en $(-2, 2)$
    Decreciente en $(-\infty, -2) \cup (2, \infty)$
14. Punto mínimo $\left(0, -\frac{1}{4}\right)$
    Creciente en $(0, 2) \cup (2, \infty)$
    Decreciente en $(-\infty, -2) \cup (-2, 0)$
15. Punto máximo $(-6, -12)$
    Punto mínimo $(0, 0)$
    Creciente en $(-\infty, -6) \cup (0, \infty)$
    Decreciente en $(-6, -3) \cup (-3, 0)$

EJERCICIO 42

1.



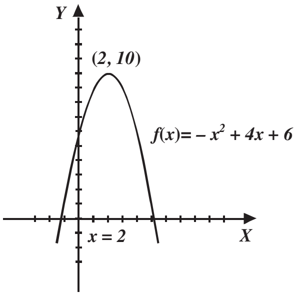
Punto mínimo (3, 1)

Crece $(3, \infty)$

Decrece $(-\infty, 3)$

Concavidad hacia arriba $(-\infty, \infty)$

2.



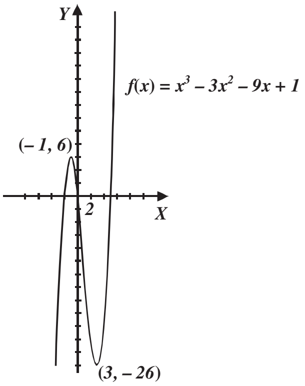
Punto máximo (2, 10)

Crece $(-\infty, 2)$

Decrece $(2, \infty)$

Concavidad hacia abajo $(-\infty, \infty)$

3.



Punto máximo $(-1, 6)$, Punto mínimo $(3, -26)$

Crece $(-\infty, -1) \cup (3, \infty)$

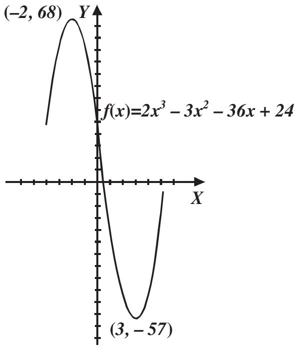
Decrece $(-1, 3)$

Concavidad hacia abajo $(-\infty, 1)$

Concavidad hacia arriba $(1, \infty)$

Punto de inflexión $(1, -10)$

4.



Punto máximo $(-2, 68)$

Punto mínimo $(3, -57)$

Crece $(-\infty, -2) \cup (3, \infty)$

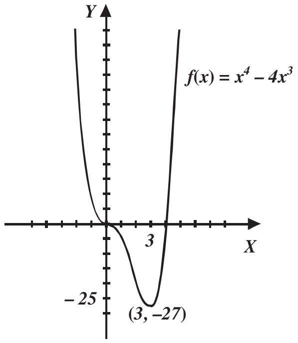
Decrece $(-2, 3)$

Concavidad hacia abajo $\left(-\infty, \frac{1}{2}\right)$

Concavidad hacia arriba $\left(\frac{1}{2}, \infty\right)$

Punto de inflexión $\left(\frac{1}{2}, \frac{11}{2}\right)$

5.



Punto mínimo $(3, -27)$

Puntos de inflexión $(0, 0)$, $(2, -16)$

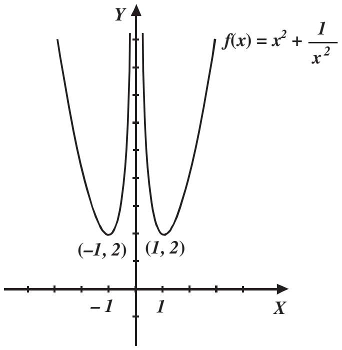
Crece $(3, \infty)$

Decrece $(-\infty, 0) \cup (0, 3)$

Concavidad hacia abajo $(0, 2)$

Concavidad hacia arriba $(-\infty, 0) \cup (2, \infty)$

6.



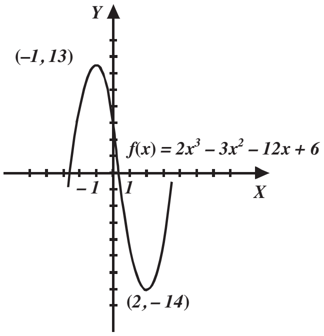
Puntos mínimos $(-1, 2), (1, 2)$

Crece $(-1, 0) \cup (1, \infty)$

Decrece $(-\infty, -1) \cup (0, 1)$

Concavidad hacia arriba $(-\infty, 0) \cup (0, \infty)$

7.



Punto máximo $(-1, 13)$

Punto mínimo $(2, -14)$

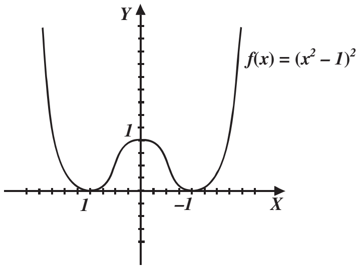
Crece $(-\infty, -1) \cup (2, \infty)$

Decrece $(-1, 2)$

Concavidad hacia abajo $\left(-\infty, \frac{1}{2}\right)$

Concavidad hacia arriba $\left(\frac{1}{2}, \infty\right)$, Punto de inflexión $\left(\frac{1}{2}, -\frac{1}{2}\right)$

8.



Punto máximo $(0, 1)$

Puntos mínimos $(-1, 0), (1, 0)$

Crece $(-1, 0) \cup (1, \infty)$

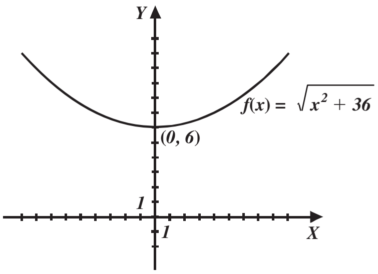
Decrece $(-\infty, -1) \cup (0, 1)$

Concavidad hacia abajo $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Concavidad hacia arriba $\left(-\infty, -\frac{1}{\sqrt{3}}\right) \cup \left(\frac{1}{\sqrt{3}}, \infty\right)$

Punto de inflexión $\left(-\frac{1}{\sqrt{3}}, \frac{4}{9}\right), \left(\frac{1}{\sqrt{3}}, \frac{4}{9}\right)$

9.



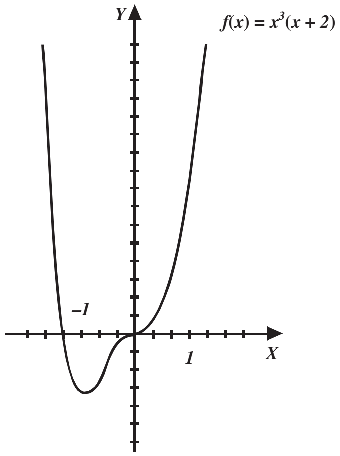
Punto mínimo $(0, 6)$

Crece $(0, \infty)$

Decrece $(-\infty, 0)$

Concavidad hacia arriba $(-\infty, \infty)$

10.



Punto mínimo $\left(-\frac{3}{2}, -\frac{27}{16}\right)$

Puntos de inflexión $(0, 0), (-1, -1)$

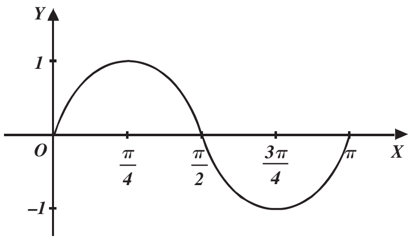
Crece $\left(-\frac{3}{2}, 0\right) \cup (0, \infty)$

Decrece $\left(-\infty, -\frac{3}{2}\right)$

Concavidad hacia abajo $(-1, 0)$

Concavidad hacia arriba $(-\infty, -1) \cup (0, \infty)$

11.



Punto mínimo $\left(\frac{3\pi}{4}, -1\right)$

Punto máximo $\left(\frac{\pi}{4}, 1\right)$

Punto de inflexión $\left(\frac{\pi}{2}, 0\right)$

Decrece $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

Crece $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{3\pi}{4}, \pi\right)$

Concavidad hacia arriba $\left(\frac{\pi}{2}, \pi\right)$

Concavidad hacia abajo $\left(0, \frac{\pi}{2}\right)$

EJERCICIO 43

1. 20 y 20
2. -25 y 25
3. 2 pulgadas por lado y el volumen de 128 ln^3
4. $V = 6144\pi \text{ cm}^3$
5. $V = \frac{500\sqrt{6\pi}}{9\pi} \text{ in}^3$
6. $r = \sqrt[4]{\frac{100}{3}}$

   $h = \sqrt[4]{\frac{400}{3}}$
7. $A = 54 \text{ cm}^2$
8. $A = 24u^2$
9. Número = 1
10. Base = $\sqrt{3}\, r$

    altura = $\frac{3}{2} r$
11. $(1, 1), (-1, -1)$
12. $A = \frac{32\sqrt{3}}{9} u^2$
13. Base = $2\sqrt{2}$

    altura = $\sqrt{2}$
14. $2\sqrt{6}$ y $2\sqrt{3}$ ft
15. $d = 2\sqrt{5}$
16. 8 y 8
17. $2\sqrt{5}$ y $\sqrt{5}$ unidades
18. $A = 6u^2$
19. $h = 2$ cm
20. Cada lado mide $2u$
21. 20 y 20
22. $4\sqrt{2}$ y 8
23. $P\left(\frac{3}{2}, \frac{1}{2}\right)$
24. $A = 15u^2$
25. $A = 2ab\, u^2$
26. $4x + 3y - 24 = 0$
27. Radio $4\sqrt{2}$

    altura = $8\sqrt{2}$ pulgadas
28. $\left(\frac{1}{2}\sqrt{4a^2 + k^2}, \frac{k}{2}\right)$
29. Números 4 y 4
30. $\frac{2P}{4+\pi}; \frac{P}{4+\pi}$
31. 400 m, 800 m
32. $\frac{100\sqrt{3}\pi}{9+\sqrt{3}\pi}$ cm, $\frac{900}{9+\sqrt{3}\pi}$ cm
33. $\frac{2000\sqrt{3}\pi}{27} \text{ cm}^3$
34. $25\sqrt{2}$ cm × $25\sqrt{2}$ cm
35. $r = \frac{5}{\sqrt{\pi}}$ in = $\frac{10}{\sqrt{\pi}}$ in
36. $5\sqrt{5}$ m
37. $r = \sqrt[3]{\frac{3V}{5\pi}}$
38. $P(2, 0)$,

    $P\left(-\frac{\sqrt{6}+2}{2}, \sqrt{6} - \frac{3}{2}\right)$

EJERCICIO 44

1. $6\frac{\text{m}}{\text{s}}$, $\frac{17}{2}\frac{\text{m}}{\text{s}}$, $10\frac{\text{m}}{\text{s}}$
2. $0 < t < 4$ y $6 < t$
3. *a*) $s = 22$ si $t = 2$, $s = 18$ si $t = 4$

   $a = -6$ si $t = 2$, $a = 6$ si $t = 4$

   b) $s = 20$, $v = -3$ si $t = 3$

   c) "s" crece cuando $0 < t < 2$ o $t > 4$

   d) "v" crece cuando $0 < t < 2$ o $t > 4$
4. *a*) $t = 18$ s, $v = -54\frac{\text{m}}{\text{s}}$

   b) $t = 9$ s, $s = 243$ m
5. $v = -2\frac{\text{m}}{\text{s}}$, $s = 35$ m

EJERCICIO 45

1. $\frac{5}{2}\sqrt{10}$ cm
2. $\frac{3}{5}\sqrt{449}\frac{\text{m}}{\text{min}}$
3. $\frac{360}{\sqrt{17}}\frac{\text{m}}{\text{s}}$
4. $\frac{4}{75}\sqrt{\frac{5}{\pi}} \text{ m}^3$
5. $-\frac{25}{12}\frac{\text{m}}{\text{s}}$
6. $10\pi\frac{\text{cm}^2}{\text{s}}$
7. $-\frac{3}{49\pi}\frac{\text{m}}{\text{min}}$
8. $-\frac{405}{8\sqrt{14}}\frac{\text{km}}{\text{h}}$

9. *a*) $100\,\dfrac{\text{km}}{\text{h}}$

   b) $\dfrac{820}{\sqrt{73}}\,\dfrac{\text{km}}{\text{h}}$

10. $-\dfrac{4}{9\pi}\,\dfrac{\text{m}}{\text{min}}$

11. 90.58 km/h

12. $4.8\,\dfrac{\text{m}}{\text{s}}$

13. $\dfrac{7}{4\pi}\,\dfrac{\text{pies}}{\text{min}}$

14. $\dfrac{\sqrt[4]{27}}{5}\,\dfrac{\text{cm}}{\text{min}}$

15. $\dfrac{14}{3}\,\dfrac{u}{s}$

16. $\dfrac{7}{50}\sqrt{29}\,\dfrac{\text{m}}{\text{s}}$

17. $1.95\,\dfrac{\text{m}}{\text{s}}$

    $.4342\,\dfrac{\text{m}}{\text{s}}$

Ejercicio 46

1. *a*) $I = \$15\,275.00$, $U = \$8\,370.00$, $Q = \$47.90$
   b) $I = \$9\,424.00$, $U = \$7\,208.00$, $Q = \$26.93$
   $Q = \$16.00$ por artículo
2. Costo promedio mínimo = \$14.80 por artículo
   Se deben producir 1 225 artículos para un costo mínimo
3. Ingreso real: $I(31) - I(30) = \$156.00$
   Ingreso aproximado: \$160.00
4. 59 metros
5. $p(x) = 100 - \dfrac{1}{800}x$
   \$50.00 por boleto

Ejercicio 47

1. $\dfrac{15}{2}$
2. 2
3. -1
4. $\dfrac{1}{3}\ln 2$
5. 1
6. $-\dfrac{9}{4}$
7. 1
8. $\dfrac{1}{3}$
9. e
10. 0
11. 1
12. $-\dfrac{1}{2}$
13. $e^{-\frac{1}{2}}$
14. 0
15. e
16. ∞
17. -1
18. 1
19. 1
20. 0

Ejercicio 48

1. $c = 0$
2. $c = \dfrac{3}{4}$
3. $c = \dfrac{5}{2}$
4. $c = \dfrac{3}{4}$
5. $c = \pm\sqrt{3}$
6. $c = 0.36$
7. $c = \pm\sqrt{\dfrac{13}{3}}$
8. $c = 0$
9. $c = 0$
10. $c = \pi$
11. No es continua en $x = 1$
12. $c = 1$

Ejercicio 49

1. $c = \dfrac{3}{2}$
2. $c = \dfrac{1}{2}$
3. $c = \sqrt{3}$
4. $c = 0$
5. $c = 3$
6. $c = -1$
7. $c = 1.7613$
8. $c = 2.1750$
9. $c = 0.5413$
10. $c = 1.3204$

Ejercicio 50

1. $dy = a\,dx$
2. $dy = (2ax + b)dx$
3. $df(x) = (3x^2 - 4x)dx$
4. $ds = \left(\dfrac{1}{2\sqrt{t}} - \dfrac{1}{3\sqrt[3]{t^2}}\right)dt$
5. $dh(t) = -36t(5 - 3t^2)^5\,dt$
6. $dy = -\dfrac{6x}{(x^2-2)^5}$
7. $dy = -\dfrac{1}{x^2}\sqrt[3]{\left(\dfrac{x}{2x+3}\right)^2}\,dx$
8. $dy = \dfrac{2(x^2+1)}{\sqrt{x^2+2}}\,dx$
9. $df(x) = (7x + 5)(x - 1)^2(x + 3)^3\,dx$
10. $dh(s) = \dfrac{8}{(2s+3)^2}\,ds$
11. $dg(x) = \dfrac{-2x}{(x^2-1)^2}\,dx$
12. $dy = \dfrac{x+8}{2(x+3)^{\frac{3}{2}}}\,dx$
13. $dy = \dfrac{-2abx}{(ax^2-b)\sqrt{a^2x^4-b^2}}\,dx$
14. $df(x) = (1 + 2\,\text{sen}\,2x)dx$
15. $df(t) = 6\tan^2 2t\sec^2 2t\,dt$
16. $dy = -2\tan x(\sec x - \sec^2 x)dx$
17. $dg(x) = \dfrac{-2\cos x}{(1+\text{sen}\,x)^2}\,dx$

18. $ds(t) = \dfrac{-t \operatorname{sen} t - 2\cos t}{2t^2\sqrt{\cos t}}\,dt$

19. $df(x) = \dfrac{\sec x}{\sec x + 1}\,dx = \dfrac{1}{1+\cos x}\,dx$

20. $dy = \dfrac{2x}{x^2+5}\log e\,dx$

21. $dy = \dfrac{x}{x^2-3}\,dx$

22. $dy = \dfrac{3}{2x^2x + 32x - 4}\,dx$

23. $dy = \dfrac{3}{2}\sqrt{x}e^{\sqrt{x^3}}\,dx$

24. $dy = 2^{x^3+5}(3x^2 \ln 2)\,dx$

25. $dh(t) = -\dfrac{2}{(e^t - e^{-t})^2}\,dt$

26. $df(x) = x(\ln x^2 + 1)\,dx$

27. $df(x) = -\dfrac{2}{\sqrt{1-4x^2}}\,dx$

28. $dy = -\dfrac{2}{x^2+4}\,dx$

29. $dy = \dfrac{1}{2x\sqrt{x-1}}\,dx$

30. $dy = -\dfrac{3}{x\sqrt{9x^6-1}}\,dx$

EJERCICIO 51

1. $\approx \dfrac{167}{18} = 9.277$

2. $\approx \dfrac{89}{27} = 3.296$

3. $\approx \dfrac{17}{8} = 2.125$

4. $\approx \dfrac{45 + 2\sqrt{3}\pi}{90} = 0.620$

5. $\approx \dfrac{108 + \sqrt{3}\pi}{36} = 3.151$

6. $\approx \dfrac{76 + 30\sqrt{3} + 2\pi}{15} = 8.949$

7. $\approx \dfrac{5489}{4} = 1372.25$

8. $\approx \dfrac{180\left(\sqrt{3}-\sqrt{2}\right) - \pi\left(7+\sqrt{2}\right)}{360} = 0.8549$

9. $\approx \dfrac{45 - 3\pi}{720} = 0.054$

10. $\approx \dfrac{9 - \sqrt{3}\pi}{9\sqrt{3}} = 0.228$

11. $dA = 0.286$ cm^2

12. $dA = 2.265$ cm^2, $dV = 3.341$ cm^3

13. $dV = 1.8\pi$ cm^3

14. Lado = 8 cm

15. Error relativo = $\dfrac{dA}{A} = 0.00249$,

    Error porcentual = 0.249%

    Error relativo = $\dfrac{dV}{V} = 0.00374$,

    Error porcentual = 0.374%

16. Lado = $\dfrac{1}{9}$ cm

17. $d\phi = 0.02$ cm

18. Error relativo = $\dfrac{dA}{A} = 0.00088$,

    Error porcentual = 0.088%